

REAL ANALYSIS II

MULTIPLE CHOICE QUESTIONS

UNIT 1:

1. The function f is continuous at $a \in M$ if $\lim_{x \rightarrow a} f(x) = \text{-----}$
 - (a) $f(b)$
 - (b) $f(c)$
 - (c) $f(a)$ *
 - (d) $f(x)$
2. The open ball of radius r about a is defined by _____
 - (a) $B[r, a]$
 - (b) $B[a, r]$ *
 - (c) $B[-r, a]$
 - (d) $B[-a, r]$
3. $\{X_n\}_{n=1}^{\infty}$ is a sequence in M_1 such that $\lim_{n \rightarrow \infty} X_n = \text{_____}$
 - (a) $-a$
 - (b) o
 - (c) 1
 - (d) a *
4. Every function from $\mathbb{R}^d$ is continuous on _____
 - (a) $\mathbb{R}^d$ *
 - (b) $-\mathbb{R}^d$
 - (c) $\mathbb{R}^{2d}$
 - (d) $\mathbb{R}^{3d}$
5. $\langle M, \rho \rangle$, both M and ϕ are _____
 - (a) 1
 - (b) Open sets *
 - (c) 0
 - (d) -1
6. Every subset of $\mathbb{R}^d$ is _____
 - (a) open
 - (b) closed
 - (c) Open *
 - (d) finite
7. Every open subset G of $\mathbb{R}^1$ can be written as _____
 - (a) 5
 - (b) 3
 - (c) open & closed
 - (d) $G = \bigcup I_n$ *

8. E is a closed subset of M if $E =$ _____
- (a) E^*
 - (b) W
 - (c) G
 - (d) U
9. The set $\bar{E}$ of all limit points of E is called the _____
- (a) open
 - (b) Closure of E^*
 - (c) open
 - (d) compact
10. The subset A of M is said to be dense in M if $\bar{A} =$ _____
- (a) N
 - (b) L
 - (c) M^*
 - (d) U

UNIT 2:

1. Every bounded subset of $\mathbb{R}^2$ is _____
- (a) Bounded
 - (b) Not bounded
 - (c) Totally bounded *
 - (d) None
2. Every subsequence of a convergence sequence is _____
- (a) Divergent
 - (b) Continuous
 - (c) convergent *
 - (d) Both (a) and (b)
3. Class of functions are called _____
- (a) contractions *
 - (b) Distractions
 - (c) Divergent
 - (d) Convergent
4. $\{X_n\}_{k=1}^{\infty}$ is a Cauchy subsequence of _____

- (a) ∞
- (b) $\{\{X_n\}_{n=1}^{\infty}\}^*$
- (c) 0
- (d) X_n

5. The Metric space $\langle M, \rho \rangle$ is both complete and totally bounded is said to be _____

- (a) scalar
- (b) complete
- (c) compact *
- (d) discrete

6. The space $\mathbb{R}^d$ with finite subset is _____

- (a) discrete
- (b) complete
- (c) compact *
- (d) scalar

7. If M is a compact metric space then M has a _____

- (a) Heine Borel Property *
- (b) vector
- (c) scalar
- (d) mean value theorem

8. If $F_1, F_2, \dots, F_n \in \mathcal{F}_1$ then $F_1 \cap F_2 \cap \dots \cap F_n \neq$ _____

- (a) 1
- (b) 0
- (c) 2
- (d) ϕ^*

9. The real valued function f is continuous at the point $a \in \mathbb{R}$ if given $\varepsilon > 0$ there exist $\delta > 0$ such that _____

- (a) 0
- (b) 1
- (c) $|f(x) - f(a)| < \varepsilon$ *
- (d) ϕ

10. If the real valued function f is continuous on the closed bounded interval $[a, b]$, then f is _____

- (a) Uniformly Continuous *
- (b) continuous
- (c) convergent
- (d) divergent

UNIT 3:

1. If χ is a characteristic function of rational numbers $[0,1]$ then for any interval $J \subset [0,1]$ then $m[\chi, J] =$ _____

- (a) 0 *
- (b) ∞
- (c) $-\infty$
- (d) 1

2. If f is bounded function on the closed bounded interval $[a,b]$, we say that f is _____

- (a) Riemann – integral *
- (b) Continuous
- (c) Bounded
- (d) None

3. $\int_a^b f(x).dx = \int_{-b}^{-a} f(x).dx =$ _____

- (a) 1 *
- (b) 0
- (c) ∞
- (d) $-\infty$

4. $\int_a^b f(x).dx =$ _____

- (a) $\text{lub } U(f, \sigma)$
- (b) $\text{glb } U(f, \sigma)$ *
- (c) $\text{lub } U(g, \sigma)$
- (d) $\text{glb } U(g, \sigma)$

5. $\bigcup_{m=1}^{\infty} E_m =$ _____

- (a) E *
- (b) M
- (c) E_n
- (d) 0

6. $E^*m \cup E^{**}m = \underline{\hspace{2cm}}$

- (a) E^*
- (b) E^*
- (c) $\cap E^*$
- (d) UE^{**}

7. $U[f, \tau] - L[f, \tau] < \underline{\hspace{2cm}}$

- (a) $d|J|$
- (b) $a|J|$
- (c) $b|J|$
- (d) $c|J|$

8. $\int_a^c f + \int_c^b f = \underline{\hspace{2cm}}$

- (a) $\int_a^b f$
- (b) $\int_a^b f$
- (c) $\int_c^b f$
- (d) $\int_d^c f$

9. If $f \in R(a, b)$ $g \in R(a, b)$ and $(f+g) \in R(a, b)$ then $\int_a (f+g) = \underline{\hspace{2cm}}$

- (a) $\int_a^b f + \int_a^b g$
- (b) $\int_a^b g + \int_a^b f$
- (c) $\int_a^b f$
- (d) $\int_a^b g$

10. Show that $f(x) = 2x+1$ is integrating on $[1,2]$ and $\int (2x+1).dx = \underline{\hspace{2cm}}$

- (a) 0
- (b) 2
- (c) 4 *
- (d) -4

UNIT 4:

1. g is continuous function defined by $g(x) = \underline{\hspace{2cm}}$

- (a) $\langle \cos x, \sin x \rangle$ *
- (b) $\langle -\cos x, \sin x \rangle$
- (c) $\langle \cos x, -\sin x \rangle$
- (d) $\langle -\cos x, -\sin x \rangle$

2. f is uniformly continuous on $\mathbb{R}^1$, if given $\varepsilon > 0$, there exists $\underline{\hspace{2cm}}$

- (a) $\delta > 0$ *
- (b) $\delta > 1$
- (c) $\delta < 1$
- (d) $\delta < 0$

3. Subset B of $\mathbb{R}^2$ consisting of graph of $y = \sin(1/x)$, $0 < x \leq 1$ together with the $\underline{\hspace{1cm}}$ interval on y -axis from $\langle 0, -1 \rangle$ to $\langle 0, 1 \rangle$

- (a) Open
- (b) Closed *
- (c) Continuous
- (d) Both (a) and (b)

4. Subset of $\mathbb{R}^2$ is bounded, iff it is contained in some square whose edge has $\underline{\hspace{1cm}}$ length

- (a) infinite
- (b) finite *
- (c) none
- (d) both (a) and (c)

5. Subset of $\mathbb{R}^2$ is bounded ,iff it is contained in some _____ whose edge has finite length

- (a) Square*
- (b) rectangle
- (c) triangle
- (d) none

6. The interval _____ is not a bounded subset of $\mathbb{R}'$

- (a) $(0,\infty)^*$
- (b) $(\infty,0)$
- (c) $(0,1)$
- (d) $(1,\infty)$

7. Bounded and totally bounded are not at all _____

- (a) non- equivalent
- (b) Equivalent*
- (c) finite
- (d) none

8. Since $\rho(e_j, e_k) = \sqrt{2}$,if $j \neq k$, the sequence $\rho_1, \rho_2, \dots$ has no _____

- (a) Cauchy's sequence
- (b) Cauchy subsequence*
- (c) Convergent sequence
- (d) Divergent sequence

9. Metric space $[0,1]$ is _____ for $[0,1]$ is a closed subset of $\mathbb{R}'$

- (a) Compact
- (b) Connect
- (c) Complete*
- (d) None

10. Metric space is denoted by _____

- (a) $\langle M, \rho \rangle$
- (b) $\langle M, \rho \rangle^*$
- (c) $\langle m, \rho \rangle$
- (d) $\langle m, \rho \rangle$

UNIT5:

1. If f has a derivative at c and it is denoted by _____

- (a) $f(c)$
- (b) $f'(c)$ *
- (c) $f(1)$
- (d) $f'(0)$

2. Define Rolle's theorem.

3. Write down the statement of mean value theorem.

4. If f has a derivative at c then it is _____ at c .

- (a) Neither or nor continuous
- (b) Bounded
- (c) Continuous *
- (d) Both (a) and (b)

5. If f has derivative at c and g has derivative at $f(c)$ then $g \circ f$ has a _____ at c .

- (a) Compact
- (b) Complete
- (c) Connectedness
- (d) Derivative *

6. If E is any subset of a metric space M then _____

- (a) $\bar{E} \subset E$
- (b) $E \subset \bar{E}$ *
- (c) $E \supset \bar{E}$
- (d) $\bar{E} \supset E$

7. The union of a infinite number of closed sets need not be a _____

- (a) Closed set *
- (b) Open set
- (c) Both (a) and (b)
- (d) Union

8. If A and $\emptyset$ are both open and closed in metric space $\langle A, \rho \rangle$ then A is said to be _____

- (a) Complete
- (b) Compact
- (c) Connected ***
- (d) Closed

9. If a subset A of the metric space $\langle M, \rho \rangle$ is totally bounded then A is _____

- (a) Unbounded
- (b) Bounded ***
- (c) Continuous
- (d) Closed

10. The space $\mathbb{R}'$ is complete but not _____

- (a) Connect
- (b) Continuous
- (c) Compact***
- (d) None

K2 QUESTIONS:

UNIT 1:

1. If a homeomorphism from M_1 onto M_2 exist , we say that M_1 and M_2 are

Answer: Homeomorphism

2. Metric space M is totally bounded if it has _____ sets.

Answer: finite number

3. If f is continuous at a , then $\omega[f;a] =$ _____

Answer: 0

4. If f is not continuous at a , then $\omega[f;a] >$ _____

Answer: 0

5. Let $A = [0,1]$, which of the following subsets of A are open subset of A .

Answer: $(1/2,1)$

6. $\bar{A}_1 \cap A_2 =$ _____, $A_1 \cap \bar{A}_2 = \phi$

Answer: ϕ

7. x and f are both continuous then $x \circ f$ is _____

Answer: Continuous *

8. A is not bounded we write $\text{diam } A =$ _____

Answer: ∞ *

9. If $T: M \rightarrow m$ is a contraction on M then $\rho(t_x, t_y) \leq$ _____

Answer: $\alpha(\rho(x, y))$

10. If every Cauchy sequence of sequence if points in M converges to points in M is called a

Answer: Complete metric space

UNIT 2:

1. Function f is bounded if its range $f(A)$ is a _____

Answer: Bounded subset

2. If f is a real valued function on a set A that f attains a maximum value of $a \in A$ if _____

Answer: $f(a) \geq f(x), x \in A$

3. If f is a real valued function on a set A that f attains a minimum value of $a \in A$ if _____

Answer: $f(a) \leq f(x), x \in A$

4. A function is continuous if and only if it is uniformly continuous then it is said to be

Answer: Compact metric space

5. The subset E of R' is said to be _____.

Answer: Measure zero

6. $\sum_{n=1}^{\infty} E_n = \underline{\hspace{2cm}}$

Answer: Measure zero

7. $I_1 = [X_0, X_1], I_2 = [X_1, X_2], \dots, I_n = [X_{n-1}, X_n]$ are called $\underline{\hspace{2cm}}$

Answer: Component interval of σ

8. $U[f; \sigma] \geq \underline{\hspace{2cm}}$

Answer: $L(f, \sigma)$

9. $\int_a^b f(x).dx = \underline{\hspace{2cm}}$

Answer: $\text{lub } L(f, \sigma)$

10. If χ is a characteristic function of relational numbers $[0, 1]$ then for any interval $J \subset [0, 1]$ then $M[\chi, J] = \underline{\hspace{2cm}}$

Answer: 1

UNIT 3:

1. If T^* is any refinement of T , it may be show that $L[f; T] \leq \underline{\hspace{2cm}}$

Answer: $L[f; T^*]$

2. $\sum_{n=1}^{\infty} |I_n|$ converges to $\underline{\hspace{2cm}}$

Answer: $\sum_{n=1}^{\infty} E_n$

3. $\int_a^b \lambda f = \underline{\hspace{2cm}}$

Answer: $\lambda \int_a^b f$

4. The function f is defined by $f(x) = x^2 + 2x$, $0 < x < 4$, then $f(x)$ is $= \underline{\hspace{2cm}}$

Answer: 15

5. F_1 and F_2 are closed subsets of metric M , then $F_1 \cup F_2$ is _____

Answer: Closed

6. The union of an infinite number of closed sets is _____

Answer: need not be a closed set

7. The set is open if and only if its complement is _____

Answer: Closed

8. If f is continuous at a iff _____

Answer: $\lim_{n \rightarrow \infty} X_n = a$

9. M is a metric space with _____ property

(a) Heine borel

10. If $|x-a| < \delta$, then the limit exceeds from _____

Answer: $-\infty < a < \infty$

UNIT 4:

1. Metric space $\langle M, \rho \rangle$ the sets M and ϕ are both _____

Answer: Open and closed

2. If A is not bounded, then we write $\text{diam } A$ equal to _____

Answer: ∞

∞

3. $\bigcap_{n=1}^{\infty} F_n$ contains _____

Answer: One point

4. The non-empty subsets $A_1, A_2, \dots, A_n$ of M exists such that _____

Answer: $\text{diam } A_k < 1$

5. Which one is correct form of 'contradiction' _____

Answer: $\rho(tx,ty) \leq \alpha(\rho(x,y))$

6. Choose the correct example for continuity of the inverse function

Answer: $f(x)=x$

7. g is continuous function defined by _____

Answer: $g(x) = \langle \cos x, \sin x \rangle$

8. Let g be the continuous function defined by $g(x) = \langle \cos x, \sin x \rangle$, $0 \leq x \leq 2\pi$, then g^{-1} is _____

Answer: Continuous

9. f is homomorphism of _____

Answer: m_1 onto m_2

10. The space $\mathbb{R}^d$ with ∞ subset cannot be _____

Answer: Compact

UNIT 5:

1. If A is a closed subset of a compact metric space $\langle M, \rho \rangle$ then A is also _____

Answer: Compact

2. If M is a compact metric space then M has a _____ property.

Answer: Heine – Borel

3. If the metric space M has a Heine-Borel property then M is _____

Answer: Compact

4. If f has a derivative at c then it is _____ at c .

Answer: Continuous

5. If f has derivative at c and g has derivative at $f(c)$ then $g \circ f$ has a _____ at c .

Answer: Derivative

6. If E is any subset of a metric space M then _____

Answer: $E \subset \bar{E}$

7. The union of a infinite number of closed sets need not be a _____

Answer: Closed set

8. If A and ϕ are both open and closed in metric space $\langle A, \rho \rangle$ then A is said to be _____

Answer: Connected

9. If a subset A of the metric space $\langle M, \rho \rangle$ is totally bounded then A is _____

Answer: Bounded

10. The space $\mathbb{R}'$ is complete but not _____

Answer: Compact

REAL ANALYSIS II

K2 QUESTIONS:

Unit 1

1. If the real valued functions f and g are continuous at $a \in \mathbb{R}'$, then so are $f+g$, $f-g$ and fg . If $g(a) \neq 0$, then f/g is also continuous at 'a'.
2. If f and g are real valued functions, if f is continuous at a , and if g continuous at $f(a)$, then $g \circ f$ is continuous at 'a'.
3. The real valued function f is continuous at $a \in \mathbb{R}'$ iff given $\epsilon > 0$ there exist $\delta > 0$ such that $|f(x) - f(a)| < \epsilon$, $|x - a| < \delta$.
4. The real valued function f is continuous at $a \in \mathbb{R}'$ iff the inverse image under f of any open ball $B[f(a), r]$ about $f(a)$ contains a open ball $B[a, \delta]$ about a .
5. The real valued function f is continuous at $a \in \mathbb{R}'$, iff whenever $X_n \xrightarrow{\infty} a$ is the sequence of real numbers convergent to 'a'. Then the sequence $f(X_n) \xrightarrow{\infty} f(a)$ converges to $f(a)$ ie f is continuous at 'a' iff $\lim_{n \rightarrow \infty} f(X_n) = f(a)$
 $\Leftrightarrow \lim_{n \rightarrow \infty} X_n = a \Rightarrow \lim_{n \rightarrow \infty} f(X_n) = f(a)$
6. The function f is continuous at $a \in M$ if any one of the following condition holds
 - (i) Given $\epsilon > 0$ there exist $\delta > 0$ such that $\rho_2(f(x), f(a)) < \epsilon$, $\rho_1(x, a) < \delta$
 - (ii) The inverse image under f of any open ball $B[f(a), \epsilon]$ about $f(a)$ contains an open ball $B[a, \delta]$ about 'a'.
 - (iii) Whenever $X_n \xrightarrow{\infty} a$ is a sequence of points in M , converging to 'a'.

Then the sequence $f(X_n) \xrightarrow{\infty} f(a)$ of points in M_2 converges to $f(a)$.

7. Let $\langle M_1, \rho_1 \rangle, \langle M_2, \rho_2 \rangle$ be metric spaces and let $f: M_1 \rightarrow M_2$ and $g: M_2 \rightarrow M_3$. If f is continuous at $A \in M_1$ and g is continuous at $f(a) \in M_2$. Then $g(f)$ is continuous at A .
8. If f and g are continuous function from a metric space M_1 into a metric space M_2 then so are $f+g$, $f \cdot g$ and f/g further more $g(x) \neq 0$, $x \in M_1$.
9. Let $\mathcal{F}$ be a non-empty family of open subsets of a metric space M . Then $\bigcup \mathcal{F}$ is also in open subset of M .
10. If G_1 and G_2 are open subsets of metric space M then $G_1 \cap G_2$ is also an open set.

Unit 2:

1. Let $\langle M, \rho \rangle$ be a metric space and let 'A' be a proper subset of M then the subset G of A is an open subset of metric space $\langle A, \rho \rangle$ iff there exists an open subset G_M of metric space $\langle M, \rho \rangle$ such that $G_A = A \cap G_M$ (ie) A set is open in metric space $\langle A, \rho \rangle$ iff it is intersection of a set with 'A' that is open in metric space $\langle M, \rho \rangle$.
2. Let $\langle M, \rho \rangle$ be a metric space and Let A be a subset of M , then if 'a' has either one of the following properties it has the other.
 - i) Non empty subset A_1 and A_2 of M such that $A = A_1 \cup A_2$, $A' \cap A_2 = \emptyset$, $A_1 \cap A_2' = \emptyset$
 - ii) When $\langle A, \rho \rangle$ metric space then there is no set except A and $\emptyset$ which is both open and closed in metric space $\langle A, \rho \rangle$. This we say that A is connected.

3. The subset of A of R_1 is connected iff whenever $a \in A$, $b \in A$ with $a < b$ then $C \in A$ for any C , such that $a < C < b$ that is whenever $a \in A$, $b \in A$, $a < b$, then $(a, b) \subset A$.
4. Let F be a continuous function from metric space M_1 . If M_2 is connected then the range of F into connected.
5. Let M be a metric space then M is connected iff every continuous characteristic function on M , is constant c (ie) M is connected iff the function identically 'zero' and the function identically '1' are the only characteristic functions on M that are continuous on M .
6. If A_1 and A_2 be connected subsets of a metric space M and if $A_1 \cap A_2 \neq \emptyset$ then $A_1 \cup A_2$ is also connected.
7. If the subset A of the metric space $\langle M, \rho \rangle$ is totally bounded then A is bounded.
8. The subset A of the metric space $\langle M, \rho \rangle$ is totally bounded if and only if for every $\varepsilon > 0$, $A \supset$ a finite subset $\{x_1, \dots, x_n\}$ which is ε dense in A .
9. Let $\langle M, \rho \rangle$ be a metric space, the subset A of M is totally bounded iff every sequence of points of A contains a Cauchy sub sequences.

Unit 3:

1. The metric space $\langle M, \rho \rangle$ is compact iff every sequence of points in M has a subsequence converging to a point in M
2. If A is a closed subset of the compact metric space $\langle M, \rho \rangle$ then the metric space $\langle A, \rho \rangle$ is also compact
3. Let A be a subset of a metric space $\langle M, \rho \rangle$ is $\langle A, \rho \rangle$ is compact, then A is also closed subset of $\langle M, \rho \rangle$
4. If M is a compact metric space then M has a Heine Borel property
5. If the metric space M has a Heine Borel property then M is compact.
6. The metric space M is compact iff whenever $\mathcal{F}$ is a family of closed subset of M with the finite intersection property then $\bigcap_{F \in \mathcal{F}} F \neq \emptyset$
7. Let f be a continuous function from compact metric space M_1 , into the metric space M_2 , then the range of f ie $f(M_1)$ is also compact
8. Let f be a continuous function from the compact metric space $M_1 \rightarrow M_2$ then the range of $f(M_1)$ of f is a bounded subset of M_2
9. If the real valued function f is continuous on closed bounded interval in R' then f must be bounded
10. If the real valued function f is continuous on the compact metric space M then f attains a maximum value at some point of M also f attains a minimum value at some point of M .

Unit 4:

1. If each of the subset $E_1, E_2, \dots$ of R' is of measure zero, then $\bigcup_n E_n$ is also of measure zero
2. If f be a bounded function on $[a, b]$ then every upper sum for f is greater than or equal to every lower sum of f that is if σ and T are any two subdivisions of $[a, b]$ then $u[f, \sigma] \geq L[f, T]$

3. Let f be a bounded function on the closed bounded interval $[a, b]$ then $f \in R[a, b]$ if and only if f is continuous at almost every point in $[a, b]$
4. If $\omega[f; x] < a$ for each x in a closed bounded interval J then there is a subdivision $\tau(J)$ such that $U[f, \tau] - L[f, \tau] < a |J|$
5. If $f \in R[a, b]$ and $a < c < b$ then $f \in R[a, c]$, $f \in R[c, b]$ and $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
6. If $f \in R[a, b]$ and λ is any real numbers then $\lambda f \in R[a, b]$ and $\int_a^b \lambda f(x) dx = \lambda \int_a^b f(x) dx$
7. Every countable subset of R' as measure zero
8. If $f \in R(a, b)$ and λ is any real number then $\lambda f \in R(a, b)$ and $\int_a^b \lambda f(x) dx = \lambda \int_a^b f(x) dx$
9. State and prove chain rule.
10. STATE AND

Unit 5:

1. Mean value theorem or Lagrange's mean value theorem.
2. If f is a continuous real valued function on the interval J and if $f'(x) > 0$ for all x in J except possibly the end point of J then F is strictly increasing on J
3. Let f and g be continuous functions on the closed bounded interval $[a, b]$ with $g(b) \neq g(a)$ if both f and g has derivative at each point of (a, b) and $f'(t)$ and $g'(t)$ are not both equal to zero for any $c \in (a, b)$ then there exist a point $c \in (a, b)$ such that $f'(c)/g'(c) = (f(b)-f(a))/(g(b)-g(a))$
4. If f is a continuous on a closed bounded interval $[a, b]$ and if $F(x) = \int_a^x f(t) dt$ at, $a \leq x \leq b$
Then $F(x) = f(x)$, $a \leq x \leq b$
5. If the real valued function f has the derivative at the point $c \in R'$ then f is continuous at c .
6. If $f \in R[a, b]$ if $f(x) = \int_a^x f(t) dt$ at $a \leq x \leq b$ and if f is continuous at $x_0 \in [a, b]$ then $f'(x) = f(x_0)$
7. Let f be a continuous real valued function on the closed bounded interval $[a, b]$. If the maximum value for f is attained at c where $a < c < b$ and if $f'(c)$ exists then $f'(c) = 0$
8. Let f be a continuous real valued function on the closed bounded interval $[a, b]$. If the minimum value of f is attained at c where $a < c < b$ and if $f'(c)$ exists then $f'(c) = 0$
9. If $f'(x) = 0$, for every x in the closed bounded interval $[a, b]$ then f is constant and closed interval $[a, b]$ $f(x) = c$, $a \leq x \leq b$ for some $c \in R$
10. IF $f'(x) = g'(x)$ for all x in the closed bounded interval $[a, b]$ when $f-g$ is constant i.e) $f(x) = g(x) + c$.

REAL ANALYSIS II

K3 QUESTIONS:

UNIT 1:

1. Every open set G of $\mathbb{R}'$ can be written $G = \bigcup I_n$ where $I_1, I_2, \dots$ are mutually disjoint open intervals.
2. Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces and let $f: M_1 \rightarrow M_2$ then f is continuous on M_1 if $f^{-1}(G)$ is open in M_1 whenever G is open in M_2 . That is f is continuous iff the inverse image of every open set is open.
3. Let E be a subset of metric space M , then the point $x \in M$ is a limit point of E iff every open ball $B[x; r]$ about x contains at least one point of E .
4. If F_1 and F_2 are closed subsets of metric M , then $F_1 \cup F_2$ is also closed.
5. If $\mathcal{F}$ is a family of closed subsets of a metric space M then intersection $\bigcap \mathcal{F}$ is a closed set.

UNIT 2:

1. If $\langle M, \rho \rangle$ is a complete metric space, A is closed subset of M then $\langle A, \rho \rangle$ is also complete.
2. Let $\langle M, \rho \rangle$ be a complete metric space for $n \in \mathbb{I}$. Let F_n be a closed bounded subset of M such that
 - (i) $F_1 \supset F_2 \supset \dots \supset F_n \supset F_{n+1} \supset \dots$
 - (ii) $\text{Diam } F_n \rightarrow 0$ as $n \rightarrow \infty$ then $\bigcap_{n=1}^{\infty} F_n$ contains precisely one point.
3. Let $\langle M, \rho \rangle$ be a complete metric space if T is a contraction on M then, there is one and only one point x in M such that $Tx = x$.
4. The subset A of $\mathbb{R}^d$ is totally bounded iff it contains only a finite number of points.
5. If f is a continuous real valued function on the closed bounded interval $[a, b]$ then f takes on every value between $f(a)$ and $f(b)$.

UNIT 3:

1. If the real valued function f is continuous on the closed bounded interval $[a, b]$ then f attains the maximum or minimum values at point $[a, b]$.
2. If f is a one to one continuous function from the compact metric space m_1 onto the metric space m_2 , then f^{-1} is continuous on m_2 and hence f is a homeomorphism of m_1 onto m_2 .
3. Let $\langle M, \rho_1 \rangle$ be a compact metric space if f is a continuous function from M_1 into M_2 a metric space $\langle M_2, \rho_2 \rangle$ then f is uniformly continuous on m_1 .
4. If the real valued function is continuous on the closed bounded interval $[a, b]$ then f is uniformly continuous on $[a, b]$.

- Let $\langle M_1, \rho_1 \rangle$ be a metric space and let A be a dense subset of M_1 . If f is a uniformly continuous from $\langle A, \rho_1 \rangle$ into a complete metric space $\langle M_2, \rho_2 \rangle$ then f can be extended to a uniformly continuous function f from M_1 into M_2

UNIT 4:

- If $f \in R(a, b)$ $g \in R(a, b)$ and $(f+g) \in R(a, b)$ then $\int (f+g) = \int f + \int g$
- Properties of the Rie-mann integral
- If $f \in R[a, b]$, $g \in R[a, b]$ and if $f(x) \leq g(x)$ almost everywhere ($a \leq x \leq b$) then $\int f \leq \int g$
- If $f \in R[a, b]$ then $|f| \in R[a, b]$ and $|\int f| \leq \int |f|$

5. Let f be a bounded function on the closed bounded interval $[a, b]$ then $f \in R[a, b]$ if and only if for each $\epsilon > 0$ there exist a subdivision $\sigma([a, b])$ such that $U(f, \sigma) < L(f, \sigma) + \epsilon$

UNIT 5:

- Second fundamental theorem of calculus
- If f and g both have derivatives at $c \in R'$ then $f+g$, $f-g$, fg also have derivatives at $c \in R'$ and $(f+g)' = f'(c) + g'(c)$ and $(f-g)' = f'(c) - g'(c)$ and $(fg)' = f'(c) \cdot g(c) + f(c) \cdot g'(c)$ further more if $g'(c) \neq 0$, then f/g has a derivatives at c and $(f/g)' = (g(c)f'(c) - f(c)g'(c)) / [g(c)]^2$
- Suppose f has a derivative of c and g has a derivative at $f(c)$, then $\phi = g \circ f$ has a derivative at c and $\phi'(c) = g'(f(c))f'(c)$
- Let ϕ be a real valued function on closed bounded interval $[a, b]$ such that ϕ' is continuous on $[a, b]$. Let $A = \phi(a)$ $B = \phi(b)$ then if f is continuous on $\phi[a, b]$ we have $\int_A^B f(x) dx = \int_a^b f(\phi(x)) \phi'(x) dx$
- If f has a derivative at every point of $[a, b]$. Then f' takes an every value between $f'(a)$ and $f'(b)$