

REAL ANALYSIS I
K1-LEVEL QUESTIONS
UNIT I

1. Set is a collection of well defined -----objects.
 - a. **Distinct**
 - b. same
 - c. Equal
 - d. None
2. The objects in a set are called its -----.
 - a. objects
 - b. **Elements**
 - c. both a and d are true
 - d. None
3. In the set $A = \{a, b, c, d\}$, which statement is true?
 - a. e belongs to A
 - b. c belongs to A
 - c. a belongs to A
 - d. **both b and c are true**
4. As a set which has no element is called -----
 - a. infinite set
 - b. **Empty set**
 - c. universal set
 - d. None
5. The Cartesian product of A and B is the set of all -----where $a \in A$ and $b \in B$.
 - a. triplet
 - b. single element
 - c. **Ordered pairs**
 - d. None
6. In a function definition, $y = f(x)$, y is called -----
 - a. **Image of x under f**
 - b. range
 - c. domain
 - d. None
7. In a function f from A into B, the set A is called -----of f.
 - a. range

- b. element
 - c. object
 - d. Domain**
8. Composition of functions satisfy -----law.
- a. Associative**
 - b. identity
 - c. absorption
 - d. None
9. If $f(x) = f(y)$ implies $x = y$, then the function is called -----
- a. One to One**
 - b. Many to one
 - c. onto
 - d. None
10. If there exists one to one correspondence between the sets A and B, then A and B are called -----
- a. Equivalent**
 - b. not equivalent
 - c. neither a nor b
 - d. either a or b

Unit II

11. In sequence s_i ($i=1,2,\dots$) is called ----- .
- a. i^{th} term of the sequence.**
 - b. n^{th} term of the sequence.
 - c. j^{th} term of the sequence.
 - d. none
12. Which one is the sequence of the subsequence $\{2,3,5,7,\dots\}$.
- a. $\{1,2,3,\dots\}$**
 - b. $\{0,1,2,3,\dots\}$
 - c. $\{2,3,5,7,11,\dots\}$
 - d. none
13. $\{1,3,5,\dots\}$ is the subsequence of -----.
- a. $\{1,2,3,\dots\}$**
 - b. $\{0,1,2,3,\dots\}$
 - c. $\{2,3,5,7,11,\dots\}$
 - d. none
14. Which one is the subsequence of $\{1,0,1,0,\dots\}$?
- a. $\{1,1,1,1,\dots\}$**

- b. $\{0,0,0,0,\dots\}$
 - c. both a and b**
 - d. neither a nor b
15. The sequence $\{S_n\}$ has the limit L , then -----.
- a. $|S_n - L| < \epsilon$
 - b. $|S_n + L| < \epsilon$
 - c. $|S_n - L| < \epsilon$
 - d. None
16. In $\{1, \frac{1}{2}, \frac{1}{3}, \dots\}$, then the limit of the sequence is -----
- a. 0**
 - b. 1
 - c. 2
 - d. None
17. If a sequence of nonnegative numbers, then its limit-----
- a. $L \leq 0$
 - b. $L \geq 0$**
 - c. $L < 0$
 - d. None
18. If the sequence of real numbers has the limit L , then we say that the sequence is -----.
- a. Convergent**
 - b. Divergent
 - c. Oscillatory
 - d. None
19. A sequence can not converge to ----- limit.
- a. More than one**
 - b. More than two
 - c. same
 - d. None
20. Which one is convergent sequence?
- a. $\{+1, -1, +1, -1, \dots\}$
 - b. $\{1, 2, 3, \dots\}$
 - c. $\{1, 1, 1, 1, \dots\}$**
 - d. None

Unit III

21. The limit of the sum of two convergent sequence is -----.
- a. $L + M$**

b. $L - M$

c. LM

d. L/M

22. If $\lim_{n \rightarrow \infty} n$ then $\lim_{n \rightarrow \infty} n$?

a. **cL**

b. L

c. both a and b

d. None

23. If $0 < x < 1$, then x^n converges to -----.

a. 1

b. **0**

c. -1

d. None

24. The limit of the difference of two convergent sequence is -----.

a. $L + M$

b. **$L - M$**

c. LM

d. L/M

25. The limit of the product of two convergent sequence is -----.

a. $L + M$

b. $L - M$

c. **LM**

d. L/M

26. The limit of the quotient of two convergent sequence is -----.

a. $L + M$

b. $L - M$

c. LM

d. **L/M**

27. $\lim_{n \rightarrow \infty} \frac{n}{n} = ?$

a. 1

b. **2**

c. 0

d. None

28. If two sequences are diverge to infinity then their sum = ?

a. Converge to L

b. **Diverge to infinity**

c. both a and b

- d. None
29. If two sequences diverge to infinity then their product = ?
- Converge to L
 - Diverge to infinity**
 - Oscillatory
 - None
30. If $\sum_{n=1}^{\infty} a_n$ diverges to infinity and $\sum_{n=1}^{\infty} b_n$ converges, then their sum is --
- Diverge to infinity**
 - Convergent
 - oscillatory
 - None

Unit IV

31. ----- is an ordered pair $\langle \sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n \rangle$.
- The series
 - The sequence
 - $\sum_{n=1}^{\infty} a_n$
 - both a and c**
32. In series, $a_1 + a_2 + \dots + a_n + \dots$, $a_n \in \mathbb{R}$, the number a_n is called-----
- The j^{th} term
 - The i^{th} term
 - The n^{th} term**
 - None
33. The number ----- is called the n^{th} partial sum of the series.
- $\sum_{k=1}^n a_k$
 - $\sum_{k=1}^n a_k$**
 - $\sum_{k=1}^n b_k$
 - None
34. If n is odd, what is the n^{th} partial sum of the series $1 - 1 + \dots - 1^n + \dots$?
- 1**
 - 1
 - 0
 - None
35. If n is even, what is the n^{th} partial sum of the series $1 - 1 + \dots - 1^n + \dots$?
- 1
 - 0**

- c. -1
 - d. None
36. If the sequence converges to A then the corresponding series is -----
- a. Converges to B
 - b. Diverges
 - c. Converges to A**
 - d. either a or b
37. If the sequence diverges then the corresponding series is -----.
- a. Diverges**
 - b. Converges
 - c. either a or b
 - d. neither a or b
38. The series $\sum_n 1/n$ is -----.
- a. Convergent
 - b. Divergent
 - c. Both a and b
 - d. either a or b
39. Which one is an alternate series ?
- a. $\sum_n (-1)^n$
 - b. $\sum_n (-1)^{n/n}$
 - c. Both a and b
 - d. Either a or b
40. If $\sum_n |a_n|$ converges then we say that $\sum_n a_n$ -----.
- a. Converges conditionally
 - b. Converges absolutely**
 - c. Both a and b
 - d. Either a or b

Unit V

41. In metric spaces, f be a real valued function whose domain includes all points in some open interval ----- except possibly the point a itself.
- a. $(a - h, a + h)$**
 - b. $(a + h, a + h)$
 - c. $(a - h, a - h)$
 - d. None
42. $\lim_{x \rightarrow a} f(x) = L$, then the number is called -----.

a. Right hand limit of f at a

b. Left hand limit of f at a

c. both a and b

d. none

43. $\lim_{x \rightarrow a} f(x) = L$, then the number is called -----.

a. Left hand limit of f at a

b. limit of f at a

c. Right hand limit of f at a

d. None

44. If both the left and right hand limit exists and equal to L, then it is called -
-----.

a. Limit of f.

b. Limit of a

c. Limit of b

d. none

45. If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$ then $\lim_{x \rightarrow a} (f(x) + g(x)) = ?$

a. L

b. F

c. A

d. None

46. What is the condition for the real valued function f is nondecreasing on J?

a. $f(x) \geq f(y)$ whenever $x > y$; $x, y \in J$

b. $f(x) \leq f(y)$ whenever $x < y$; $x, y \in J$

c. $f(x) \leq f(y)$ whenever $x = y$; $x, y \in J$

d. $f(x) \geq f(y)$ whenever $x < y$; $x, y \in J$

47. What is the condition for the real valued function f is nonincreasing on J?

a. $f(x) \geq f(y)$ whenever $x < y$; $x, y \in J$

b. $f(x) \leq f(y)$ whenever $x < y$; $x, y \in J$

c. $f(x) \leq f(y)$ whenever $x = y$; $x, y \in J$

d. $f(x) \leq f(y)$ whenever $x > y$; $x, y \in J$

48. What is the condition for the real valued function f is monotone on J?

a. $f(x) \leq f(y)$ whenever $x < y$; $x, y \in J$ and $f(x) \geq f(y)$ whenever $x < y$; $x, y \in J$

b. $f(x) \leq f(y)$ whenever $x > y$; $x, y \in J$ and $f(x) \geq f(y)$ whenever $x > y$; $x, y \in J$

c. $f(x) \geq f(y)$ whenever $x < y$; $x, y \in J$ and $f(x) \leq f(y)$ whenever $x < y$; $x, y \in J$

d. None

49. If the left hand limit exists then we say that f is ----- on (a,b).

a. Bounded above

b. Bounded below

c. Bounded

d. None

50.If the right hand limit exists then we say that f is ----- on (a,b) .

a. Bounded above

b. Bounded below

c. Bounded

d. None

K2-LEVEL QUESTIONS

UNIT I

1. A set A is said to be ----- if A is equivalent to the set I of positive integers.

Countable

2. ----- is the another name for the Countable set.

Denumerable

3. A set which is not countable is called -----

Uncountable

4. The set of all integers is -----

countable

5. The set of all ----- is uncountable.

Real numbers

6. The Countable union of countable set is -----.

Countable

7. The set of all ----- is countable.

Rational numbers

8. The glb of $(7,8)$ is -----.

7

9. The lub of $(7, 8)$ is -----.

8

10. If A is any nonempty subset of R that is bounded above, then A has a -----

LUB

Unit II

11. Write one example for divergent sequence?

$\{+1, -1, +1, -1, \dots\}$

12. If the sequence of real numbers convergent to L , then it's any subsequence is convergent to -----.

L

13. ----- of a convergent sequence of real numbers converge to the same limit.

All subsequences

14. If the limit of a sequence is infinity, then the sequence is -----
--.

Divergent

15. In divergent sequence, the limit of the sequence approaches to -----

Infinity or minus infinity

16. Write one example for a divergent sequence?

n $\frac{\infty}{n}$

17. Which subsequence of the sequence $\{1, -2, 3, -4, 5, -6, \dots\}$ is divergent to minus infinity?

$\{-2, -4, -6, \dots\}$ and $\{1, 3, 5, \dots\}$

18. The sequence neither diverge to minus infinity nor infinity then the sequence is called -----

Oscillates

19. If the sequence real numbers is convergent then it is -----.

Bounded

20. ----- is a sequence which is either nonincreasing or nondecreasing or both.

Monotone sequence

Unit III

21. What is the sum of $\{0, 1, 0, 2, 0, 3, \dots\}$ and $\{1, 0, 2, 0, 3, 0, \dots\}$?
 $\{1, 1, 2, 2, 3, 3, \dots\}$

22. The sum of $\{1, 0, 1, 0, 1, 0, \dots\}$ and $\{0, 1, 0, 1, 0, 1, \dots\}$ is a -----
-- sequence.

Convergent

23. The limit superior of the sequence $\{1, -1, 1, -2, 1, -3, 1, -4, \dots\}$ is --

1

24. In a convergent sequence, limit superior of the sequence is equal to

Limit of the sequence

25. In ----- sequence Limit superior of the sequence is equal to Limit of the sequence.

Convergent

26. $\lim_{n \rightarrow \infty} \inf - 1 = ?$

--

27. In a convergent sequence limit inferior of the sequence is equal to -

a. Limit of the sequence

28. In ----- sequence Limit inferior of the sequence is equal to Limit of the sequence.

Convergent

29. If the sequence of real numbers converges then that sequence is a --

Cauchy sequence

30. Every Cauchy sequence is a ----- sequence.

Convergent.

Unit IV

31. If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges then we say that $\sum_{n=1}^{\infty} a_n$ -----.

Converges conditionally

32. Which one is conditionally convergent series?

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

33. If we rearranging an -----series then the series remains the same.

Absolutely convergent

34. If we rearranging an -----series then the rearrangement of that series is converges.

Conditionally convergent

35. If a series converges absolutely then its rearrangement is -----.
Absolutely convergent

36. If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ then-----.
 $|a_n| \leq |b_n|$

37. If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} b_n$ converges absolutely then ----- converges absolutely.

$$\sum_{n=1}^{\infty} a_n$$

38. If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} b_n$ converges absolutely then $\sum_{n=1}^{\infty} a_n$ converges absolutely. This test is called -----
-----.

Comparison test

39. ----- test is used to test the convergence of the series $\sum_{n=1}^{\infty} \sqrt[n]{|a_n|}$.

Root test

40. If $|x| < 1$ then $\sum_{n=1}^{\infty} n^{10,000} x^n$ is -----.

Converges absolutely

Unit V

41. What is the condition for the real valued function f is strictly increasing on J ?
 $f(x) < f(y)$ whenever $x < y ; x, y \in J$
42. What is the condition for the real valued function f is strictly decreasing on J ?
 $f(x) > f(y)$ whenever $x < y ; x, y \in J$
43. In a metric space $\langle M, \rho \rangle$, ρ is called ----- on M .
Metric
44. From the given below, which one is a metric space?
a. $|x - y|$
b. $x - y$
45. If ρ is a metric for a set M then 2ρ is -----.
Metric
46. If ρ and σ are two metrics for a set M , then $\sigma + \rho$ is -----.
Metric
47. In a metric space, a convergent sequence of points of M , then the corresponding sequence is a ----- sequence.
Cauchy
48. A sequence of points in any metric space cannot converge to ----- distinct limits.
Two
49. In some metric spaces there are ----- sequences which are not convergent.
Cauchy
50. A sequence converges in $\langle M, \rho \rangle$ if and only if it converges in $\langle M, \sigma \rangle$ and the limits are same then the two metrics are-----.
Equivalent

K3- LEVEL QUESTIONS

UNIT 1

1. If $f : A \rightarrow B$ and if $X \subset B, Y \subset B$, then $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$. In words, the inverse image of the
2. If $f : A \rightarrow B$ and if $X \subset B, Y \subset B$, then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$. In words, the inverse image of the union of two sets is the union of the inverse images.
3. If $A_1, A_2, \dots$ are countable sets, then $\bigcup_{n=1}^{\infty} A_n$ is countable. In words, the countable union of countable sets is countable.
4. If $f : A \rightarrow B, g : B \rightarrow C, h : C \rightarrow D$, prove that $h \circ (g \circ f) = (h \circ g) \circ f$.
5. Let $f(x) = 2x$ ($-\infty < x < \infty$). Can you think of function g and h which satisfy the two equation

$$g \circ f = 2gh, h \circ f = h^2 - g^2 ?$$

6. The set of all rational numbers in $[0, 1]$ is countable.
7. If B is an infinite subset of the countable set A , then B is countable.
8. The set of all rational numbers is countable.
9. Which of the following define a 1-1 function?
 - (a) $f(x) = e^x$ ($-\infty < x < \infty$),
 - (b) $f(x) = \sin x$ ($-\infty < x < \infty$),
 - (c) $f(x) = ax + b$ ($-\infty < x < \infty$), $a, b \in \mathbb{R}$.
10. If A is any nonempty subset of $\mathbb{R}$ that is bounded below, then A has a greatest lower bound in $\mathbb{R}$.

UNIT 2

1. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non negative numbers and if $\lim_{n \rightarrow \infty} S_n = L$, then $L \geq 0$

2. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then $\{s_n\}_{n=1}^{\infty}$ cannot also converge to a limit distinct from L. That is, if $\lim_{n \rightarrow \infty} s_n = L$ and $\lim_{n \rightarrow \infty} s_n = M$, then $L = M$.
3. If the sequence of real number $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then any subsequence of $\{s_n\}_{n=1}^{\infty}$ is also convergent to L.
4. All subsequence of convergent sequence of real numbers converge to the same element.
5. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then $\{s_n\}_{n=1}^{\infty}$ is bounded.
6. A non decreasing sequence which is bounded above is convergent.
7. The sequence $1 - \frac{n}{n}$ is convergent.
8. A non decreasing sequence which is not bounded above diverges to infinity.
9. A non increasing sequence which is bounded below is convergent. A non increasing sequence which is not bounded below diverges to minus infinity.
10. Prove $\lim_{n \rightarrow \infty} 2n/(n+3) = 2$.

UNIT-3

1. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n \rightarrow \infty} s_n = L$, and if $\lim_{n \rightarrow \infty} t_n = M$, then $\lim_{n \rightarrow \infty} (s_n + t_n) = L + M$. In words, the limit of the sum (of two convergent sequences) is the sum of the limits.
2. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $c \in \mathbb{R}$, and if $\lim_{n \rightarrow \infty} s_n = L$, then $\lim_{n \rightarrow \infty} cs_n = cL$.
3. (a) If $0 < x < 1$, then $\{x^n\}_{n=1}^{\infty}$ converges to 0. (b) If $1 < x < \infty$ then $\{x^n\}_{n=1}^{\infty}$ diverges to infinity.
4. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequence of real numbers, if $\lim_{n \rightarrow \infty} s_n = L$, and if $\lim_{n \rightarrow \infty} t_n = M$, then $\lim_{n \rightarrow \infty} (s_n - t_n) = L - M$.
5. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are convergent sequence of real numbers, if $s_n \leq t_n$ ($n \in \mathbb{I}$), and if $\lim_{n \rightarrow \infty} s_n = L$, $\lim_{n \rightarrow \infty} t_n = M$, then $L \leq M$.

6. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, then $\{s_n\}_{n=1}^{\infty}$ converges to L^2 .
7. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n \rightarrow \infty} s_n = L$, and if $\lim_{n \rightarrow \infty} t_n = M$, then $\lim_{n \rightarrow \infty} s_n t_n = LM$.
8. If $\{t_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $\lim_{n \rightarrow \infty} t_n = M$ where $M \neq 0$, then $\lim_{n \rightarrow \infty} 1/t_n = 1/M$.
9. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n \rightarrow \infty} s_n = L$, and if $\lim_{n \rightarrow \infty} t_n = M$ where $M \neq 0$, then $\lim_{n \rightarrow \infty} s_n/t_n = L/M$.
10. Prove $\lim_{n \rightarrow \infty} \frac{n}{n} = 1$.

UNIT-4

1. If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then $\lim_{n \rightarrow \infty} a_n = 0$.
2. If $\sum_{n=1}^{\infty} a_n$ is a series of non negative numbers with $s_n = a_1 + \dots + a_n$ ($n \geq 1$) then
 - (a) $\sum_{n=1}^{\infty} a_n$ converges if the sequence $\{s_n\}_{n=1}^{\infty}$ is bounded; (b) $\sum_{n=1}^{\infty} a_n$ diverges if $\{s_n\}_{n=1}^{\infty}$ is not bounded.
3. If $\sum_{n=1}^{\infty} a_n$ converges to A and If $\sum_{n=1}^{\infty} b_n$ converges to B, then $\sum_{n=1}^{\infty} (a_n + b_n)$ converges to A+B. Also, if $c \in \mathbb{R}$, then $\sum_{n=1}^{\infty} c a_n$ converges to c A.
4. (a) If $0 < x < 1$, then $\sum_{n=1}^{\infty} x^n$ converges to $1/(1-x)$. (b) If $x \geq 1$, then $\sum_{n=1}^{\infty} x^n$ diverges.
5. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.
6. If $\sum_{n=1}^{\infty} a_n$ is a convergent series of positive numbers, then there is a sequence $\{\epsilon_n\}_{n=1}^{\infty}$ of positive numbers which converges to zero but for which $\sum_{n=1}^{\infty} \epsilon_n a_n$ still diverges.
7. If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that
 - (a) $a_1 \geq a_2 \geq \dots \geq a_n \geq a_{n+1} \geq \dots$
 - (b) $\lim_{n \rightarrow \infty} a_n = 0$, then the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent.
8. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.

9.(a) . If $\sum_n a_n$ converges absolutely then both $\sum_n a_n$ and $\sum_n |a_n|$ converge. however, (b) . If $\sum_n a_n$ converges conditionally, then both $\sum_n a_n$ and $\sum_n |a_n|$ diverge.

10. Let $\sum_n a_n$ be a conditionally convergent series of real numbers. Then for any $x \in \mathbb{R}$ there is a rearrangement of $\sum_n a_n$ which converges to x .

UNIT-5

1. If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then $f(x)+g(x)$ has a limit as $x \rightarrow a$ and, in fact, $\lim_{x \rightarrow a} [f(x) + g(x)] = L + M$.

2. If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$, then

(a) $\lim_{x \rightarrow a} [-g(x)] = -M$

(b) $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = L \cdot M$

And if $M \neq 0$,

(c) $\lim_{x \rightarrow a} [f(x)/g(x)] = L/M$.

3. Let f be a non decreasing function on the bounded open interval (a,b) . If f is bounded above on (a,b) , then $\lim_{x \rightarrow a^+} f(x)$ exists. Also, if f is bounded below on (a,b) then $\lim_{x \rightarrow b^-} f(x)$ exists.

4. Let f be a non increasing function on the bounded open interval (a,b) . If f is bounded above on (a,b) , then $\lim_{x \rightarrow a^+} f(x)$ exists. Also, if f is bounded below on (a,b) then $\lim_{x \rightarrow b^-} f(x)$ exists.

5. If f is a monotone function on the open interval (a,b) , and if $c \in (a,b)$, then $\lim_{x \rightarrow c^-} f(x)$ and $\lim_{x \rightarrow c^+} f(x)$ both exists.

6. Let $\langle M, \rho \rangle$ be a metric space and let 'a' be a point in M . Let f and g be real-valued functions whose domains are subsets of M . If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = N$, then

$$\lim_{x \rightarrow a} [f(x) + g(x)] = L + N,$$

$$\lim_{x \rightarrow a} [-g(x)] = -N,$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = LN.$$

And, if $N \neq 0$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n g(x_k) = L/N.$$

7. Let $\langle M, \rho \rangle$ be a metric space. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points of M . Then $\{s_n\}_{n=1}^{\infty}$ is a Cauchy.

8. Define metric spaces with example.

9. prove that $d: \mathbb{R} \times \mathbb{R} \rightarrow [0, \infty)$ BY

$$d(x, x) = 0 \quad x \in \mathbb{R},$$

$$d(x, y) = 1, \quad x, y \in \mathbb{R}; \quad x \neq y \quad \text{is a metric spaces.}$$

10. Show that if ρ is a metric for a set M then so is 2ρ .

K4- LEVEL QUESTIONS

UNIT I

- Find the glb and lub for the following sets.
(a) (7,8) (b). [1,2] (c). [1,3] (d). (- 1,1]
- If A is any nonempty subset of R that is bounded above, then A has a least upper bound in R.
- If A,B are subset of S, then $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
- Prove, for any sets A, B, C, that $(A \cap B) \cap C = A \cap (B \cap C)$
- Prove $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

UNIT II

- Prove that $\{\sqrt{n}\}_n^\infty$ diverges to infinity.
- let $\{s_n\}_n^\infty$ be the sequence defined by

$$S_1=1$$

$$S_2=2$$

$$S_{n+1}=s_n+s_{n-1} \quad (n=3,4,5,\dots). \text{ Find } s_8.$$

- Define convergent sequence with example.
- Explain monotone sequence with example.
- Explain the bounded sequence with example.

UNIT III

- If $\{s_n\}_n$ and $\{t_n\}_n$ are sequences of real numbers that diverge to infinity, then so do their sum and product. That is, $\{s_n+t_n\}_n$ and $\{s_n t_n\}_n$ diverge to infinity.
- If $\{s_n\}_n$ and $\{t_n\}_n$ are sequences of real numbers, If $\{s_n\}_n$ diverges to infinity, and if $\{t_n\}_n$ is bounded, then $\{s_n+t_n\}_n$ diverges to infinity.
- If $\{s_n\}_n$ diverges to infinity and if $\{t_n\}_n$ converges, then $\{s_n+t_n\}_n$ diverges to infinity.
- If $\{s_n\}_n$ is a convergent sequence of real numbers, then $\lim_{n \rightarrow \infty} \sup s_n = \lim_{n \rightarrow \infty} s_n$

15. If $\{s_n\}$ is a convergent sequence of real numbers, then
 $\liminf_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} s_n$.
16. If $\{s_n\}$ is a Cauchy sequence of real numbers, if and only if $\{s_n\}$ is a convergent.

UNIT IV

17. If $\sum_n a_n$ converges absolutely to A, then any rearrangement $\sum_n b_n$ of $\sum_n a_n$ also converges absolutely to A.
18. If the series $\sum_n a_n$ and $\sum_n b_n$ converge absolutely to A and B, respectively, then $AB = C$ where $C = \sum_n c_n$ and $c_n = \sum_{k=0}^n a_k b_{n-k}$ ($k=0,1,2,3,\dots$)
19. If $\sum_n a_n$ is dominated by $\sum_n b_n$ and $\sum_n |b_n| = \infty$, then $\sum_n |a_n| = \infty$.
20. If $\sum_n a_n$ is dominated by $\sum_n b_n$ where $\sum_n b_n$ converges absolutely, then $\sum_n a_n$ also converges absolutely. Symbolically, if $\sum_n a_n \ll \sum_n b_n$ and $\sum_n |b_n| < \infty$, then $\sum_n |a_n| < \infty$.
21. If $\limsup_{n \rightarrow \infty} \sqrt[n]{|a_n|} = A$ then the series of real numbers $\sum_n a_n$ (a) converges absolutely if $A < 1$, (b) diverges if $A > 1$

UNIT V

22. Show that if ρ and σ are both metric for a set M, then $\rho + \sigma$ is also a metric for M.
23. For $P = \langle x_1, y_1 \rangle$ and $Q = \langle x_2, y_2 \rangle$, define

$$\sigma(P, Q) = |x_1 - x_2| + |y_1 - y_2|.$$

Show that σ is a metric for set of ordered pairs of real numbers.

Also, if

$$\tau(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|),$$

Show that τ defines a metric for a same set.

24. Show that a sequence of points in any metric spaces cannot converge to two distinct limits.
25. Show that $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$.
26. Prove

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0.$$

