K1 LEVEL QUESTIONS

UNIT I

1. The set of -------- with usual definitions of addition and multiplication is a field
    a. Real Numbers
    b. Complex numbers
    c. Only a
    d. **Both a and b**
2. The set of ------- with usual definitions of addition and multiplication is a field.
    a. Complex Numbers
    b. Natural numbers
    c. Only b
    d. Both a and b
3. A map F:VxV into V is called an ----------composition in V.
    a. internal binary
    b. external binary
    c. only b
    d. both a and b
4. A map g: VxV into V is called an ----------composition in V.
    a. external binary
    b. internal binary
    c. both a and b
    d. none
5. ---------- matrix is an example of row reduced matrix.
    a. Identity
    b. Null
    c. Diagonal
    d. square
6. Every mxn matrix over the field F is ---------------- to row reduced matrix.
    a. row equivalent
    b. not equivalent
    c. both a and b
    d. none
7. If left inverse and right inverse are exist then it is called -------
    a. Two-sided inverse.
    b. Left inverse
    c. Right inverse
    d. None
8. A product of invertible matrices is -------
    a. Invertible
    b. Equivalent
    c. Only b
    d. Both a and b
9. An ---------- matrix is invertible.
    a. Elementary Matrix

b. Equivalent
c. Both a and b
d. None

10. What are all the solutions to the system $x - 2y = 3$ and $-2x + 4y = -6$?
    a. x = 1 and y =-1
    b. x = 1 and y = 3
    c. x = 2 and y = 2
    d. x = - 1 and y = 2
11. Solutions to the system $x + 2y = 3$ and $3x + y = 4$?
    a. x = - 1 and y = 2
    b. x = 1 and y = 1
    c. x = - 1 and y =1
    d. x = - 1 and y = 3
12. What are all the solutions to the system $x + 2y = 3$ and $2x + 4y = 5$?
    a. x = 1 and y = -1
    b. x = 1 and y = 4
    c. no solution
    d. x = 1 and y = 1
13. If AX = 0 then the system of equations is called------
    a. Homogeneous
    b. Non-homogeneous
    c. Nonlinear
    d. None
14. Equivalent system of linear equations have exactly ---------solutions
    a. Same
    b. Different
    c. Trivial
    d. None
15. If the equations are homogeneous then the system is ----------
    a. AX= 0
    b. AX = Y
    c. AX not equal to 0
    d. None
16. -------system of equations of linear equations have exactly same solutions.
    a. Non equivalent
    b. Equivalent
    c. Homogeneous
    d. None

Unit II

17. If V is finite dimensional and W is a subspaces of V. Then dim(V/W) =-----
    a. dim V – dim W
    b. dim V + dim W
    c. dim V
    d. dim W
18. Which of the following is not a vector space?
    a. R(C).
    b. R(R)

c. C(C)
d. None

19. In a vector space V, the elements of V are called --------
    a. **Vectors**
    b. Scalars
    c. Both a and b
    d. None
20. In a vector space V, the elements of F are called -------------
    a. **Scalars**
    b. Vectors
    c. Both a and b
    d. None
21. The zero vector space is also called as ----------
    a. **Null vector space**
    b. Unit vector space
    c. Both a and b
    d. None
22. The linear span of S is defined as the set consisting of all -----------of finite subsets of the element of S.
    a. **linear combinations**
    b. linear span
    c. linear equations
    d. none of these
23. The ----------- of S is defined as the set consisting of all linear combinations of finite subsets of the element of S.
    a. **Linear span**.
    b. Linear combinations
    c. Linear equations
    d. None
24. The collection of all cosets of W in V by V/W, then V/W is called ------
    a. **Quotient space**
    b. Hyperspace
    c. Both a and b
    d. None
25. a0 = ----
    a. **0**
    b. 1
    c. 2
    d. -1
26. The intersection of two subspace W1 and W2 of a vector space is also ----- of V.
    a. **Subspace**
    b. Dual space
    c. Quotient space
    d. None
27. If 0 belongs to S and contained in V, then S is------------
    a. **Linearly dependent**
    b. Independent

c. Only b
d. None

28. Intersection of two linearly independent sets of vectors is --------
    a. **Linearly independent**.
    b. Dependent
    c. Only b
    d. None
29. Union of two linearly dependent sets of vectors is ----------
    a. **Linearly dependent**.
    b. Independent
    c. Both a and b
    d. None
30. A basis of a vector space does not contain ---------
    a. **zero vector**
    b. unit vector
    c. identity vector
    d. none
31. If dim V = n, then the number of vectors in a basis of V is = -------
    a. **n**
    b. –n
    c. n+1
    d. n-1
32. A superset of a dependent vectors is linearly ------------
    a. **dependent.**
    b. Independent
    c. Both a and b
    d. None
33. A subset of a set of independent vectors is ---------------
    a. **independent**
    b. dependent
    c. only b
    d. none
34. Any subset containing ------vectors, of an n-dimensional vector space is linearly dependent
    a. **n+1**
    b. n-1
    c. n
    d. none
35. Basis of the trivial space is the ----- set.
    a. **Empty**
    b. Nonempty
    c. Whole set
    d. Universal set
36. The vector space $R^n(R)$ is --------- dimensional.
    a. **Finite**
    b. Infinite

c. Only b
d. None of these

37. Each finitely generated vector space has a ----------
   a. Finite Basis
   b. Infinite basis
   c. Empty basis
   d. None
38. The basis {(1,0,0),(0,1,0),(0,0,1)} is known as the ------basis of R^3(R).
   a. Standard
   b. Empty
   c. Nonempty
   d. None
39. If W is a subspace of an n-dimensional vector space, then--------
   a. dim W < n
   b. dim W = n
   c. dim W > n
   d. none
40. The number of elements in any basis of vector space R^4(R) is -----------
   a. 3
   b. 2
   c. 4
   d. 0
41. V is a vector space, M is its subspace dim V = n and dim(V/M) = r then dim M is ----------
   a. n + r
   b. n – r
   c. n
   d. r
42. Number of elements in a basis of a finitely generated vector space is called -------- of a vector space.
   a. Dimension
   b. Basis
   c. Zero
   d. None
43. If V and W are finite dimensional spaces over the same field F then dim(VxW) = ----------
   a. **dim V + dim W.**
   b. dim V - dim W
   c. dim V/ dim W
   d. none
44. The number of elements in any basis of the vector space R^3(R) is ------
   a. 3
   b. 2
   c. 4
   d. 1

Unit III

45. If T: V into W is the zero transformation the Range (T) = -------
   a. {0}

b. {1}
c. {2}
d. {1,2}

46. If T:V into W is the identity map, then Nullity T = --------
    a. 0
    b. 1
    c. 2
    d. 3
47. If T:V into W is the identity map then Rank T= --------
    a. dim V
    b. dim W
    c. dim (V,W)
    d. none
48. If T: V into W is a linear transformation such that dim V = 3 Nullity T = 1, then Rank T= ----------
    a. 2
    b. 3
    c. 4
    d. 1
49. If T1 and T2 are invertible linear transformations then $(T1T2)^{-1}$ is equal to--------
    a. $T1^{-1}T2^{-1}$
    b. $T1^{-1}$
    c. $T2^{-1}$
    d. None
50. If T: Vinto W then nullspace T =--------
    a. {0}
    b. {2}
    c. {1}
    d. None
51. A linear transformation is also known as -----
    a. Vector space homomorphism
    b. Isomorphism
    c. Vector space isomorphism
    d. None
52. If T:V into W is a linear transformation , then T(cx+y) = -------
    a. cT(x)+T(y)
    b. cT(x)-T(y)
    c. cT(x)/T(y)
    d. none
53. Under linear transformation zero is mapped to----------
    a. Zero
    b. One
    c. Two
    d. None
54. Dimension of null space (T) = ----------
    a. Nullity (T)

b. Rank (T)
c. Range(T)
d. None

55. Dimension of range of T = ---------------

a. Rank(T)
b. Nullity(T)
c. Null space(T)
d. None

56. If T:V into W is a linear transformation, then null space (T) is a subspace of -----------

a. V
b. W
c. Only b
d. Both a and b

57. If T:V into W is a linear transformation, then range (T) is a subspace of -----------

a. W
b. V
c. Both a and b
d. None

58. If T:V into W is a linear transformation, then Rank (T) + nullity(T) = ---------

a. dim V
b. dim W
c. dim U
d. none

59. An invertible linear transformation is also known as a ----------

a. Vector space homomorphism
b. Homomorphism
c. vector space isomorphism
d. none

60. $((T^{-1}))^{-1}$ = ------

a. T
b. V
c. W
d. None

61. If V is a vector space over the field F such that dim V = n then V is isomorphic to ---------

a. F^n(F)
b. F^n
c. W
d. none

62. Sum of two linear transformations is a -------

a. linear transformation
b. linear functional
c. linear equations
d. none

63. ---------of an invertible linear transformation is an invertible linear transformation.

a. Inverse
b. Transpose

c. Adjoint
d. None

64. The identity map on a vector space is a ----------
    a. identity transformation
    b. zero transformation
    c. linear transformation
    d. none

65. F:R^3 into R^2 whose image is generated by (1,-1,2,3) and (2, 3, -1,0) --------
    a. (x+2y, -x+3y, 2x-y,3x)
    b. (x-2y, -x-3y, 2x-y,3x)
    c. (x,-x+3y, 2x,3x)
    d. None

Unit IV

66. The transpose of the matrix of coefficients is called the ----------- matrix P.
    a. Transition
    b. Unit
    c. Null
    d. None

67. If B = $P^{-1}AP$ then B is -----------to A.
    a. Similar
    b. Nonsimilar
    c. Null
    d. None

68. The relation of similarity is an -------- relation.
    a. Equivalence
    b. non equivalence
    c. linear
    d. none

69. Determinant of T is the determinant of -------- relative to any ordered basis of V.
    a. matrix of T
    b. transformation
    c. inverse
    d. none

70. Similar matrices have the ---------
    a. Same determinant
    b. Different determinant
    c. Singular value
    d. None

71. The only matrix similar to identity matrix is -----------
    a. identity matrix
    b. null
    c. diagonal
    d. both b and c

72. If T and S are similar linear transformations on a finite dimensional vector space then det T = -----------

a. **det S**
b. det T
c. det V
d. det W

73. Transition matrix from one basis to the other is -----------
   a. **non-singular**
   b. singular
   c. zero
   d. one

74. Matrix of a linear transformation T: R^3 into R^2 relative to any basis is of order ------------
   a. **3x2 or 2x3**
   b. 2x2
   c. 3x3
   d. 4x4

75. The matrix A defined by --------is called the matrix of T relative to the pair of ordered bases $\mathfrak{B}\ and\ \mathfrak{B}$.
   a. **A(i,j) =A_{ij}**
   b. A_{ij}
   c. A(i,j)
   d. none

76. If V is a finite dimensional vector space and V^* is its dual space then
   a. dim V = dim V^*
   b. dim V > dim V^*
   c. dim V < dim V^*
   d. None

77. If S1 and S2 are subsets of a vector space V(F) such that S1 contained in S2 then,
   a. $S1^0 \subset S2^0$
   b. $S2^0 \subset S1^0$
   c. $S1^0 \quad S2^0$
   d. None

78. If W is a subspace of a vector space V(F) such that dim V = 8 and dim W= 5 then dim W^0 =---
   a. 13
   b. 8
   c. 3
   d. 5

79. If W_1 and W_2 are subspaces of a vector space V(F) such that V =$W \oplus W$, then $W \oplus W$ =-------
   a. V
   b. V^*

c. V^{**}
d. None

80. If W is a subspace of a finite dimensional vector space V(F), then the annihilator of annihilator of W is --------

a. W
b. V
c. U
d. W^0

Unit V

81. If S is a subset of a vector space V(F), then the annihilator S^0 of S is a subspace of ------

a. V^*
b. V
c. V^{**}
d. None

82. If W is a subspace of a finite dimensional vector space V(F) and W^0 is the annihilator of W then dim W = --------

a. dim V – dim W^0
b. dim V +dim W^0
c. dim V / dim W^0
d. none

83. Homomorphism of (V,F) is called -------- space of V .

a. Dual
b. Double dual
c. Second dual
d. None

84. Homomorphism of (V,F) is called dual space of V it is denoted by -------

a. V^*
b. V
c. V^{**}
d. None

85. If V is finite dimensional then dim V^* ------ to dim V^{**}

a. Equal
b. Unequal
c. Neutral
d. None

86. If V is a vector space, then V^{**} is known as ---------

a. **second dual space**
b. dual
c. inverse
d. none

87. Each linear functional is a-----------
    a. linear Transformation
    b. nonlinear
    c. annihilator
    d. none

88. The property that the second dual space V^* of a finite dimensional vector space V over a field F is isomorphic to V through the natural isomorphism from V to V^* is called ------
    a. Reflexivity
    b. Convexity
    c. Concave
    d. None

89. If S is a subset of a vector space V, then $[L(S)]^0$ =----------
    a. $\mathbf{S^0}$
    b. S^1
    c. S_0
    d. None

90. For a finite dimensional vector space V the space V and its second dual V^{**} are said to be --------spaces.
    a. Reflexive
    b. Transitive
    c. Symmetric
    d. Anti-symmetric

91. Homomorphism of (V,W) consists of all ----------from V to W.
    a. Linear transformation
    b. Duals
    c. Second duals
    d. None

92. If W is a subspace of a finite dimensional vector space V and W^0 is the annihilator of W then dim W = -------------
    a. $\mathbf{\dim V - \dim W^0}$
    b. dim V+ dim W^0
    c. dim V/ dim W^0

  - **d.** none

93. If V is finite dimensional vector space then V is ----------- to V^*.
    - **a. Isomorphic**
    - **b. Not isomorphic**
    - **c. Not homomorphism**
    - **d. Both b and c**
94. Inverse of the transpose of an invertible linear transformation T is the----------
    - **a. $(T^{-1})^t$.**
    - b. T^{-1}
    - c. $(T)^t$
    - d. None
95. The rank of transpose of linear transformations T:V into W = -------
    - **a. Rank(T).**
    - b. Range (T)
    - c. Null space(T)
    - d. None
96. A -------- in V is a maximal proper subspace of V.
    - a. **Hyperspace**
    - b. Subspace
    - c. Null space
    - d. None
97. For each linear transformation T: V into W there is a --------linear transformation T^t : W^* into V^*.
    - **a. Unique**
    - b. Different
    - c. Two
    - d. None
98. Rank (T) = ---------
    - **a. Rank (T^t)**
    - b. Rank T
    - c. Nullity (T)
    - d. None
99. The null space of T^t is ----------------- of the range of T.
    - a. **Annihilator**
    - b. Dual
    - c. Double dual

d. Second dual

100. The range of T^t is the annihilator of the ------------ of T.

a. Null space

b. Range

c. Dual

d. Linear functional

LINEAR ALGEBRA

K2 LEVEL QUESTIONS

UNIT I

1. Give an example of row reduced matrix.

   Identity Matrix
2. Define two-Sided inverse.

   If left inverse and right inverse are exist then it is called Two-sided inverse.
3. What are all the solutions to the system $x - 2y = 3$ and $-2x + 4y = -6$?

   x = 1 and y = 2
4. Solutions to the system $x + 2y = 3$ and $3x + y = 4$?

   x = 1 and y = 1.
5. What are all the solutions to the system $x + 2y = 3$ and 2x + 4y = 6?

   x = 1 and y = 1.
6. Define Homogeneous system of equations.

   If AX = 0 then the system of equations is called Homogeneous equations
7. How to get an equivalent system of equations?

   If each equation in each system is a linear combination of the equations in the other system then it is called equivalent system of equations.
8. What is the first non-zero entry in each non-zero row of row-reduced matrix.

   1
9. Give an example for row reduced Echelon matrix.

   Identity matrix
10. Write any one elementary row operations on an matrix A.

    Interchange of two rows of A

Unit II

11. If V is finite dimensional and W is a subspaces of V. Then dim(V/W) ?

    dim V – dim W
12. Write an example for not a vector space?

    R(C).
13. In a vector space V, What is the name for the elements of V and the elements of F

    Vectors and Scalars
14. What is another name for the zero vector space?

    Null vector space
15. Define Linear span.

    The linear span of S is defined as the set consisting of all linear combinations of finite subsets of the element of S.
16. Define Quotient space.

    The collection of all cosets of W in V by V/W, then V/W is called Quotient space
17. Say True or False; Any subset containing n+1 vectors of an n-dimensional vector space is linearly dependent.

    True
18. Say True or False: Basis of the trivial space is the empty set.

    True

19. The basis {(1,0,0),(0,1,0),(0,0,1)} is known as:
The standard basis of R^3(R).

20. The number of elements in any basis of vector space R^4(R):
4

Unit III

21. In T: V into W, what is the range of the zero transformation?
{0}

22. In identity map,Nullity T =?
0

23. If T:V into W is the identity map then Rank T= ?
dim V

24. If T: V into W is a linear transformation such that dim V = 3 Nullity T = 1, then Rank T=?
2

25. If T1 and T2 are invertible linear transformations then $(T1T2)^{-1}$ =?
$T1^{-1}T2^{-1}$

26. Write another name for a linear transformation.
Vector space homomorphism

27. If T:V into W is a linear transformation , then T(cx+y) = ?
cT(x)+T(y)

28. Define nullity(T).
Dimension of null space (T) is called the nullity (T)

29. Define Rank(T).
Dimension of range of T is called the rank(T)

30. If T:V into W is a linear transformation, then Rank (T) + nullity(T) = ?
dim V

31. Write another name for an invertible linear transformation.
vector space isomorphism

Unit IV

32. Define similar matrix.
If B = P^{-1}AP then B is similar to A.

33. Say True or False: The relation of similarity is an equivalence relation.
True

34. Say true or False: Similar matrices have the Same determinant
True

35. Which matrix similar to identity matrix?
identity matrix

36. If T and S are similar linear transformations on a finite dimensional vector space then what about the determinant?
det T = det S

37. Say True or False:Transition matrix from one basis to the other is non-singular.
True

38. If V is a finite dimensional vector space and V^* is its dual space then what about their dimension?

dim V = dim V^*

39. If S1 and S2 are subsets of a vector space V(F) such that S1 contained in S2 then what about their annihilators?

$S1^0 \subset S2^0$

40. If W_1 and W_2 are subspaces of a vector space V(F) such that V =$W \oplus W$, then V^* =?

$\oplus$

41. Say True or False: $(W^0)^0$ = W.

True

Unit V

42. If S is a subset of a vector space V(F), then the annihilator S^0 of S?

A subspace of V^*

43. If W is a subspace of a finite dimensional vector space V(F) and W^0 is the annihilator of W then dim W = ?

dim V – dim W^0

44. Say True or False: Homomorphism of (V,F) is called dual space of V .

True

45. Say True or False: If V is finite dimensional then dim V^* = dim V^{**}

True

46. How to denote the second dual space?

V^{**}

47. Say True or False: Each linear functional is a Linear Transformation.

True

48. Define Reflexivity property for second dual space.

The property that the second dual space V^* of a finite dimensional vector space V over a field F is isomorphic to V through the natural isomorphism from V to V^* is called reflexivity.

49. If S is a subset of a vector space V, then $[L(S)]^0$ =?

S^0

50. What is the reflexive space for second dual V^{**} ?

The vector space V

51. What is the notation for the inverse of the transpose of an invertible linear transformation T ?

$(T^{-1})^t$

52. Define Hyperspace.

A hyperspace in V is a maximal proper subspace of V.

53. Say True or False: Rank (T) = Rank (T^t)

True

K3 LEVEL QUESTIONS

UNIT I

1. Equivalent systems of linear equations have exactly same solutions
2. To each elementary row operations e there corresponds an elementary row operations e_1, of the same type as e, such that $e_1(e(A)) = e(e_1(A)= A$ for each A.
3. If A and B are row-equivalent m x n matrices, the homogeneous systems of linear equations AX =0 and BX = 0 have exactly the same solutions.
4. If A is an m x n matrix and m < n, then the homogeneous system of linear equations AX = 0 has a nontrivial solution.
5. If A is an n x n matrix, then A is row-equivalent to the n x n identity matrix if and only if the system of equation AX = 0 has only the trivial solution.
6. If A, B,C are matrices over the field F such that the product BC and A(BC) are defined, then so are the products AB, (AB)C and A(BC) =(AB)C.
7. Let A and B be m x n matrices over the field F. then B is row-equivalent to A if and only if B = PA, where P is a product of m x m elementary matrices.
8. If A has a left inverse B and a right inverse C then B = C.
9. Let A and B be n x n matrices over F.
   a. If A is a invertible so is A^{-1} and $(A^{-1})^{-1} = A$
   b. If both A and B are invertible so is AB, and $(AB)^{-1} = B^{-1} A^{-1}$
10. A product of invertible matrices is invertible.

UNIT II

11. A nonempty subsets W of V is a subspace of V if and only if for each pair of vectors α,β in W and each scalar c in F the vector $c\alpha+\beta$ is again in W.
12. If A is an m x n matrix over F and B,C are n x p matrices over F then A(dB + C) = d(AB) + AC for each scalar d in F.
13. Let V be a vector space over the field F. the intersection of any collection of subspaces of V is a subspace of V.
14. If V is a finite-dimensional vector space, then any two bases of V have the same number of elements.
15. Let S be a linearly independent subset of a vector space V. suppose β is a vector in V, which is not in the subspace spanned by S. Then the set obtained by adjoining β to S is linearly independent.
16. If W is a subspace of a finite-dimensional vector space V, every linearly independent subset of W is finite and is a part of basis for W.
17. If W is a proper subspace of a finite-dimensional vector space V, then W is finite-dimensional and dim W < dim V.
18. Let A be an n x n matrix over a field F, and suppose the row vectors of A form a linearly independent set of vectors in F^n . Then A is invertible.

19. Row-equivalent matrices have the same row-space.
20. Let R be a non-zero row-reduced echelon matrix. Then the non zero row vectors of R form a basis for the row space of R.

UNIT III

21. Let V be a finite dimensional vector space over the field F and let $\{\alpha\ , \alpha\ , \ldots , \alpha\ \}$ be an ordered basis for V. Let W be a vector space over the same field F and let , , …, be any vectors in W. Then there is precisely one linear transformation T: V into W such that Tα ; j = 1,2,….n.
22. Let T be a linear transformation from R^2 into R^3 for which T(1,2) = (3,2,1) and T(3,4) = (6,5,4). Find the formula for T.
23. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite dimensional, then rank(T) + Nullity (T) = dim V.
24. Let P be an m x m matrix and Q be a nxn matrix over F. Define a function $F^{m \times n}$ into itself by T(A) = PAQ, then prove that T is a linear transformation.
25. If A is an m x n matrix with entries in the field F, then row rank (A) = column rank (A).
26. Let V be an n-dimensional vector space over the field F, and W be an m-dimensional vector space over F. Then the space L(V, W) is finite-dimensional and has dimension mn.
27. Let V, W and Z be vector spaces over the field F. Let T be a linear transformation from V into W and U a linear transformation from W into Z. Then the composed function UT defined by (UT)(α) = U(T(α)) is a linear transformation from V into Z.
28. Let F be a field and let T be the linear operator on F^2 defined by T(x, y) = (x+y,x). Then find the explicit formula for T^{-1}.
29. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. If T is invertible, then the inverse function T^{-1} is a linear transformation from W onto V.
30. Let T be a linear transformation from V into W. Then T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

UNIT IV

31. Let V be an n-dimensional vector space over the field F and W be an m-dimensional vector space over F. Let $\mathfrak{B}$ be an ordered basis for V and $\mathfrak{B}'$ an ordered basis for W. For each linear transformation T from V into W, there is an m x n matrices A with entries in F such that $T\alpha\ _{\mathfrak{B}}$ $\alpha\ _{\mathfrak{B}}$ for every vector α in V. Furthermore, T$\longmapsto$ A is a one-one correspondence between the set of all linear transformations from V into W and the set of all m x n matrices over the field F.
32. Let V be an n-dimensional vector space over the field F and let W be an m-dimensional vector space over F. for Each pair of ordered bases $\mathfrak{B}$ and $\mathfrak{B}'$ for V and W respectively, the function which assigns to a linear transformation T its matrix relative to $\mathfrak{B}$, $\mathfrak{B}'$ is an isomorphism between the space L(V<W) and the space of all m x n matrices over the field F.

33. Let V, W and Z be finite dimensional vector spaces over the field F; let T be a linear transformations from V into W and U a linear transformations from W into Z. If $\mathfrak{B}$, $\mathfrak{B}'$ and $\mathfrak{B}''$ are ordered bases for the spaces V,W and Z respectively, if A is the matrix of T relative to the pair $\mathfrak{B}$, $\mathfrak{B}'$ and B is the matrix of U relative to the pair $\mathfrak{B}'$, $\mathfrak{B}''$ then the matrix of the composition UT relative to the pair $\mathfrak{B}$, $\mathfrak{B}''$ is the product matrix C = BA.
34. Let V be a finite dimensional vector space over the field F, and let $\mathfrak{B}$ $\alpha\ ,\alpha\ ,\ldots\alpha$ } and $\mathfrak{B}'$ $\alpha'\ ,\alpha'\ ,\ldots\alpha'$ } be ordered bases for V. suppose T is a linear operator on V. If P = [, ,… is the n x n matrix with columns P_j = α' $_{\mathfrak{B},}$ then T $_{\mathfrak{B}'}$ T $_{\mathfrak{B}}$. Alternatively, if U is the invertible operator on V defined by Uα α' , j = 1,2,…n, then T $_{\mathfrak{B}'}$ $_{\mathfrak{B}}$ T $_{\mathfrak{B}}$ $_{\mathfrak{B}}$.
35. Let V be the linear operator on R^3, the matrix of which in the standard ordered basis is $A = \begin{matrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{matrix}$. Find a basis for the range of T.
36. Let V be the linear operator on R^3, the matrix of which in the standard ordered basis is $A = \begin{matrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{matrix}$. Find a basis for the null space of T.
37. Let F be a field and let T be the operator on F^2 defined by T(x, y) = (x, 0). Find the standard ordered basis for T.
38. Prove that the trace function is a linear functional.
39. Let V be a finite dimensional vector space over the field F, and let $\mathfrak{B}$ $\alpha\ ,\alpha\ ,\ldots\alpha$ } be a basis for V. Then there is a unique dual basis $\mathfrak{B}^*$, ,… } for V^* such that α . For each linear functional f on V we have f = $\sum$ α and for each vector α in V we have α $\sum$ α α .
40. Explain about dual space and dual basis with example.

UNIT V

41. Let V be a finite dimensional vector space over the field F. For each vector α in V define $L_\alpha(f) = f(\alpha$, f in V^*. The mapping $\alpha \longmapsto L_\alpha$ is then an isomorphism of V onto V^{**}.
42. Let V be a finite dimensional vector space over the field F. If L is a linear functional on the dual space V^* of V, then there is a unique vector α in V such that L(f) = f(α for every f in V^*.
43. Let V be a finite dimensional vector space over the field F. Each basis for V^* is the dual of some basis for V.
44. If S is any subset of a finite dimensional vector space V, then $(S^0)^0$ is the subspace spanned by S.
45. If f is a non zero linear functional on the vector space V, then the null space of f is a hyperspace in V.
46. Explain about the double dual and annihilator of annihilator.
47. Every hyperspace in V is the null space of a non zero linear functional on V.

48. If f and g are linear functional on a vector space V, then g is a scalar multiple of f if and only if the null space of g contains the null space of f.
49. If f and g are linear functional on a vector space V, then g is a scalar multiple of f if and only if $f(\alpha) = 0$ implies $g(\alpha) = 0$.
50. Let V be the space of all n x n matrices over the field F and let B be a fixed n x n matrix. If T is linear operator on V defined by T(A) = AB – BA, and if f is the trace function, what is $T^t f$?

K4 LEVEL QUESTIONS

UNIT I

1. Every m x n matrix over the field F is row-equivalent to row-reduced matrix.
2. Let e be an elementary row operation and let E be the m x m elementary matrix E = e(I). then for every m x n matrix A, e(A) = EA.
3. An elementary matrix is invertible.
4. If A is an n x n matrix, the following are equivalent.

a. A is invertible
b. A is row-equivalent to the n x n identity matrix
c. A is an product of elementary matrices

5. Let A = $A\ A\ \dots A$, where $A, \dots, A$ are n x n matrices. The A is invertible if and only if A_j is invertible.

UNIT II

6. The subspace spanned by a nonempty subset S of a vector space V is the set of all linear combinations of vectors in S.
7. Let V be the vector space which is spanned by a finite set of vectors $\beta, \beta, \dots, \beta$. Then any independent set of vectors in V is finite and contains no more than m elements.
8. If W_1 and W_2 are finite dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite dimensional and dim W_1 + dim W_2 = dim($W_1 \cap W_2$) + dim ($W_1 + W_2$).
9. Let V be an n dimensional vector space over the field F, and let and ′ be two ordered bases of V. then there is a unique, necessarily invertible, n x n matrix P with entries in F such that

a. = P ′
b. = P^{-1}

For every vector α in V. the columns of P are given by P_j = j = 1,…n.

10. Suppose P is an n x n invertible matrix over F. let V be an n dimensional vector space over F, and let be an ordered basis of V. then there is a unique ordered basis ′ of V such that

a. = P ′
b. = P^{-1}

UNIT III

11. Let V and W be finite dimensional vector spaces over the field F such that dim V = dim W. If T is a linear transformation from V into W, the following are equivalent:
    i) T is invertible
    ii) T is non-singular
    iii) T is onto, that is the range of T is W.
12. Every n-dimensional vector space over the field F is isomorphic to the space F^n.

13. Let V and W be vector spaces over the field F. Let T and U be linear transformations from V into W. The function (T+U) defined by (T+U)(α) = Tα+Uα is a linear transformation from V into W. If c is any element of F, the function (cT) defined by (cT)(α) = c(Tα) is a linear transformation from V into W. The set of all linear transformations from V into W, together with the addition and scalar multiplication defined above, is a vector space over the field F.
14. Find the range, rank, null space and nullity for the zero transformation and the identity transformation on a finite dimensional space V.
15. Let V be a vector space over the field F; Let U, T_1 and T_2 be linear operators on V; let c be an element of F.
    i) IU = UI= U
    ii) $U(T_1 + T_2) = UT_1 + UT_2$; $(T_1 + T_2)U = T_1U + T_2U$;
    iii) $C(UT_1) = (cU)T_1 = U(cT_1)$

UNIT IV

16. Let = , , be the basis for C^2 defined by = (1,0,-1), =(1,1,1) = (2,2,0). Find the dual basis of .
17. Here are three linear functionals on R^4: (, , ,) = 2 2
    (, , ,) = 2
    (, , ,) = − 2 − 4 3

    Find the basis for the annihilator.

18. Let V be a finite dimensional vector space over the field F, and let W be a subspace of V. Then dim W + dim W^0 = dim V.
19. If W is a k dimensional subspace of an n-dimensional vector space V, then W is the intersection of (n – k) hyperspaces in V.
20. If W_1 and W_2 are subspaces of a finite dimensional vector space then $W_1 = W_2$ if and only if .

UNIT V

21. Let g, , ,… be linear functional on a vector space V with respective null spaces N, , ,… . Then g is a linear combination of , ,… if and only if N contains the intersection ∩ ∩…∩ .
22. Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. The null space of T^t is the annihilator of the range of T. If V and W are finite dimensional then

i) $Rank(T^t) = rank(T)$
ii) The range of T^t is the annihilator of the null space of T

23. Let V and W be finite dimensional vector spaces over the field F. Let be an ordered basis for V with dual basis * and let $'$ be an ordered basis for W with dual basis $'^*$. Let T be a linear transformation from V into W. let A be the matrix of T relative to , $'$ and let B be the matrix of T^t relative to $'^*$, *. Then $B_{ij} = A_{ij}$.
24. Let A be any m x n matrix over the field F. Then the row rank of A = the column rank of A.
25. Let F be a field and let f be the linear functional on F^2 defined by f(x,y) = ax + by. For each of the following linear operators T, let g = T^t f and find g(x,y). (a) T(x,y) = (x,0).