

Classical Algebra

K₁ Level Questions:

UNIT I

1. $(1+x)^n =$

a) $1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots$

b) $1 + \frac{x}{1!} + \frac{1 \cdot x}{1!} + \dots$

c) $1 - \frac{x}{1!} + \frac{1 \cdot x}{1!} - \dots$

d) $1 - \frac{x}{1!} - \frac{1 \cdot x}{1!} - \dots$

2. $(1-x)^{-n} =$

a) $1 + nx + \frac{1}{1!}x^2 + \dots$

b) $1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots$

c) $1 - \frac{x}{1!} - \frac{1 \cdot x}{1!} - \dots$

d) $1 - \frac{x}{1!} + \frac{1 \cdot x}{1!} - \dots$

3. $f(m) \cdot f(n) =$

a) $f(m+n)$

b) $f(m-n)$

c) $f(m/n)$

d) $f(m \cdot n)$

4. $(1-x)^n =$

a) $1 - \frac{x}{1!} - \frac{1 \cdot x}{1!} - \dots$

b) $1 - \frac{x}{1!} + \frac{1 \cdot x}{1!} - \dots$

c) $1 - \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} - \dots$

d) $1 - \frac{x}{1!} + \frac{1}{2!}x^2 - \dots$

5. $(1+x)^n =$

a) $1 - \frac{x}{1!} + \frac{1}{2!}x^2 - \dots$

b) $1 - \frac{x}{1!} + \frac{1}{2!}x^2 - \dots$

c) $1 - \frac{x}{1!} + \frac{1}{2!}x^2 - \dots$

d) $1 - \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} - \dots$

6. Sum in infinity of the series $1 + 3/4 + 3.5/4.8 + 3.5.7/4.8.12 + \dots$

a) 2 **b) $2\sqrt{3}$** c) $2\sqrt{3}$ d) $\sqrt{2}$

7. In exponential theorem the value of $\lim_{r \rightarrow \infty} u_{r+1}/u_r =$

a) 1 **b) 0** c) 2 d) 4

8. $\{f(1)\}^x =$

a) $f(x)$ b) $f(1)$ c) $f(0)$ d) 0

9. The co-efficient of x^n in the expansion of $\frac{1-x}{1-x^2} =$

a) $\frac{1}{n!}(3n^2)$ **b) $\frac{1}{n!}(1+n-3n^2)$** c) $1+n-3n^2$ d) $-1/n!$

10. In binomial theorem $u_{r+1}/u_r =$

a) $|r|$ b) $|u|$ **c) $|x|$** d) $|x/u|$

Unit II

1. $a - \sqrt{a}$ is a root of

a) $f(x)=0$ b) $f()=1$

c) $f(x)=x$ d) 0

2. The roots of $-5x^3 + 4x^2 + 8x - 8 = 0$ are

a) 1,3 **b) $1 \pm \sqrt{-1}$, 1, 2**

c) $0, 1 \pm \sqrt{2}$, 1 d) 1, 2, 3

3. The equation of the roots $\sqrt{5} + \sqrt{2}$ are

a) $x^4 - 14x^2 + 9 = 0$ b) $x^3 - 12x^2 + 1 = 0$

c) $x^4 + 2x^2 + 3 = 0$ d) $x^3 + 12x + 1 = 0$

4. $\sum \alpha_1 = \dots\dots\dots$

a) $-1/$ b) $/ 1$

c) $1/$ d) none of these

5. The general form of arithmetic progression is

a) $\alpha, ,$ **b) $- , ,$**

c) $. .$ d) $, , -$

6. The general form of geometric progression is

a) $k/r, k, kr$ b) k, r, kr

c) $k/r, r$ d) $r, k, r.k$

7. The general form of harmonic progression is

a) $-\frac{1}{1/}$ b) $\frac{1}{1/} \frac{1}{1/}$

b) $\frac{1}{1/} \frac{1}{1/}$ d) $\frac{1}{1/}$

8. $x^3 - 2 = 0$ the value of $\sum x^2 = \dots\dots\dots$

a) pq **b) $3r - pq$**

c) $1 - pq$ d) $r - pq$

9. $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in signs. The roots are $\dots\dots\dots$

a) $\pm\sqrt{-1}, 1 \pm \sqrt{-1}$ b) $\pm\sqrt{2}, \pm\sqrt{7}$

c) 2, 3, 4, $\pm\sqrt{5}$ d) 0

10. The roots of the equations $ax^4+4bx^3+6cx^2+4dx+e=0$ is

- a) $\frac{e}{a}, \frac{d}{a}, \frac{c}{a}, \frac{b}{a}$ **b)** $-\frac{e}{a}, -\frac{d}{a}, -\frac{c}{a}, -\frac{b}{a}$
c) $\frac{e}{a}, \frac{d}{a}, \frac{c}{a}, \frac{b}{a}$ d) $-\frac{e}{a}, -\frac{d}{a}, -\frac{c}{a}, -\frac{b}{a}$

UNIT III

1. Calculate the sum of the cubes of the roots of the equation $X^4+2X+3=0$

- a) -3
b) -2
c) -6
d) -7

2. Find the sum of the fifth powers of the roots of the equation $X^4-7X^2-4X-3=0$

- a) 140**
b) 141
c) 142
d) 143

3. If α, β, γ be the roots of the equation $X^3-7X+7=0$. Find $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$

- a) 3/7**
b) 7/3
c) 3
d) 7

4. Remove the fractional coefficients from the equation $X^3 + \frac{3}{2}X^2 + \frac{5}{18}X + \frac{1}{108} = 0$

- a) $X^3 + 3X^2 - X - 6 = 0$
b) $X^3 + 3X^2 - X + 6 = 0$

c) $X^3+18X^2+10X+2=0$

d) $X^3+18X^2+10X-2=0$

5. When the degree is ----- it's roots must be its own reciprocal is α to $1/\alpha$

a) odd

b) even

c) either odd or even

d) none of these

6. Change the sign of the roots of the equation $X^7+4X^5+X^3-2X^2+7X+3=0$

a) $X^7+4X^5+X^3-2X^2+7X-3=0$

b) $X^7-4X^5+X^3-2X^2+7X+3=0$

c) $X^7+4X^5-X^3-2X^2+7X+3=0$

d) $X^7+4X^5+X^3-2X^2+7X+3=0$

7. Solve the equation $X^4-10X^3+26X^2-10X+1=0$

a) $(3\pm\sqrt{5})/2, (3\pm\sqrt{2})/3$

b) $3\pm\sqrt{}$, $\sqrt{}$

c) 3, 8

d) 0

8. Find the roots of the equation $X^5-5X^3+5X^2-1=0$

a) 1, 1, 1, $(-3\pm\sqrt{})/2$

b) 1, 2, 3, 4

c) 1, -1, -1, 1

d) 1, 0, 1, 0

9. Find the sum of the fourth power of the roots of $X^3 - 2X^2 + X - 1 = 0$

a) 5

b) 10

c) 0

d) none of these

10. When the degree is ----- its roots must be its own reciprocal is α to $1/\alpha$

a) odd

b) even

c) either odd or even

d) none of these

Unit IV

1) $f(y+h) =$

a) $a_0(y+h-\alpha_1)(y+h-\alpha_2)\dots(y+h-\alpha_n)$

b) $a_1(y+h-\alpha_1)(y+h)\dots\infty$

c) $a_0(y+h)(y+h-\alpha_2)\dots$

d) none of these

2) The roots of $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_n = 0$ are

a) $\alpha, \alpha^2, \dots, \alpha_n$

b) $\alpha_1, \alpha_2, \dots, \alpha_n$

c) $\alpha, \beta, \delta, \nu$

d) $\alpha^2, \beta^2, \delta^2, \nu^2$

3) Form of quotient $f(x) =$

A) $a_0x^n + a_1x^{n-1} + \dots + a_n$

b) $a_0 + a_1 + a_2 + \dots + \infty$

c) $a_0 + x^{n-1} + \dots + \infty$

d) None of these

4) The general form of remainder $f(x) =$

a) $(x+h)Q+R$

b) $(x-h)Q+R$

c) x^2R+Q

d) $(x+h)^2Q+R$

5) The remainder of $5x^3 + 8x^2 + 8x + 12$ is divided by $(x-4)$ is

a) 364

b) 634

c) 348

d) 0

6) The remainder of $2x^6 + 3x^5 - 15x^2 + 2x - 4$ is divided by $x+5$ is

a) 48543

b) 21452

c) 21486

d) 54325

7) Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5$ by 2 the transformed equations are

a) $x^3 + 2x^2 + 3x + 1$

b) $x^4 + 3x^3 + x^2 - 4x + 1 = 0$

c) $3x^2 + 20x + 88 = 0$

d) $3x^3 + 8x^2 + 8x + 12 = 0$

8) Increase by 7 the roots of the equation $3x^4 + 7x^3 + 5x^2 + x - 2 = 0$ the transformed equation are

a) $4x^4 + 77x^3 - 720x^2 - 2786x + 4058$

b) $3x^4 - 77x^3 + 720x^2 - 2876x + 4058$

c) $5x^3 - 87x^2 + 270x + 5082$

d) $5x^3 + 87x^2 - 720x + 8535$

9) By removing the second term, $h =$

a) a_0/a_1

b) $-a_1/na_0$

c) a_1/na_0

d) none of these

10) solve the equation $x^4 + 20x^3 - 143x^2 + 430x + 462 = 0$ by removing its second term

a) $y^4 - 7y^2 + 12 = 0$

b) $(y^2 - 3)(y^2 - 4)$

c) $y = \pm\sqrt{3}, y = \pm 2$

d) all the above

Unit V

1. The notation used to represent a matrix is

a) $[]$ b) $()$ c) $\{\}$ d) $||$

2. A square matrix is one in which the numbers of rows is to number of columns

a) equal b) unequal c) less than d) greater than

3. The condition for singular matrix is

a) $|A| \neq 0$ **b) $|A| = 1$** c) $|A| = 0$ d) $|A| \neq 1$

4. Below which is diagonal matrix

a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ **c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$** d) none of these

5. The order of unit matrix

a) 0 **b) any one order** c) 2×2 d) none of these

6. The condition for non-singular matrix is

a) $|A|=0$ **b) $|A| \neq 0$** c) $|A|=2$ d) $|A|=1$

7. The transpose matrix is denoted by

a) b) c) A^T d) TA

8. Conjugate of matrix is denoted by

a) b) c) A d) all the above

9. If A is square matrix and A is the transpose of its conjugate, such a matrix is called a

a) orthogonanal matrices **b) Hermitian matrices** c) skew matrices d) none of these

10. A matrix is defined to be array

a) square b) triangular **c) rectangular** d) circular

11. In unit matrix δ_{rs} is called as

a) Kronecker delta b) Newton delta c) Hermitian delta d) none of these

12. Null is denoted by

a) 0 b) 1 c) <0 d) >0

13. $(AB)^T =$

a) $B^T A^T$ b) A^T c) B^T d) none of these

14. The inverse of matrix " A " is

a) A^T **b) A^{-1}** c) A d) none of these

15. $(A^2)^{-1} =$

a) A^{-1} b) A^2 c) $A^{-1} \cdot A^2$ d) none of these

16. $(A^T)^{-1} =$

a) A^T **b) $(A^{-1})^T$** c) A^2 d) none of these

17. The condition for Cayley Hamilton theorem

a) $[a - I] = 0$ b) $[A - \lambda I]$ c) $[A - I]$ d) none of these

18. If all the elements of a matrix A are multiplied by a constant K, then the matrix is called as

a) diagonal matrix **b) scalar matrix** c) square matrix d) all the above

19. Find the values of for a, b, c of matrix $\begin{bmatrix} 3 & -2 \\ -3 & - \end{bmatrix} = \begin{bmatrix} 2 & -7 \\ 4 & 2 \end{bmatrix}$

a) a=-1, b=-2, c=-1 b) a=1, b=0, c=0 c) a=1, b=1, c=1 d) a=0, b=0, c=0

Classical Algebra
K2 Level Questions

UNIT I

1. $\frac{e^x}{e^y} =$

ANSWER: $1 + x^2/2! + x^4/4! + \dots \infty$

2. $A + Bn + Cn(n-1) + Dn(n-1)(n-2) =$

ANSWER: $(n+1)$

3. $(1+x)^y =$

ANSWER: $e^{y \log(1+x)}$

4. $\log \frac{1}{x} =$

ANSWER: $2(x + x^3/3 + x^5/5 + \dots)$

5. $u_{n-1} =$

ANSWER: $1 + 1/2 + 1/3 + \dots 1/n - 1 - \log(n-1)$

6. The finite limit is denoted as

ANSWER:

7. Let x be the positive fraction p/q where p and q are-----

ANSWER: positive integer

8. The unit limit lying between -----

ANSWER: 0 and 1

9. $\log_e 1 =$ -----

ANSWER: 0

10. $\log(1+x) =$ -----

ANSWER: $x - \frac{x}{2} + \frac{x}{4} - \frac{x}{8} + \dots$

UNIT II

1. $\alpha\beta$ value of $ax^4+4bx^3+6cx^2+4dx+e=0$ is

ANSWER: d/b

2. What is the general form if roots are equal in magnitude and opposite in sign

ANSWER: $-$

3. The roots of $81x^3-18x^2-36x+8=0$ is

ANSWER: $2/9, 2/3, -2/3$

4. $\alpha\beta\gamma$ value of $81x^3-18x^2-36x+8=0$ is

ANSWER: $-8/81$

5. The value of $\sum \alpha^4$ in the equation $x^4+px^3+qx^2+rx+s=0$ is

ANSWER: $p^4-4p^2q+2q^2+4pr-4s$

6. The roots of $x^3+ax^2+bx+c=0$ are

ANSWER: α, β, γ

7. The equation of $\beta - \gamma - 2\alpha, \gamma - \alpha - 2\beta, \alpha - \beta - 2\gamma$ is

ANSWER: $x^3+px^2+qx+r=0$

8. The required equation in symmetric function is

ANSWER: $x^3 - x^2 + x - 1 = 0$

9. The value of $\alpha \alpha \alpha \dots \alpha =$

ANSWER: -1

10. In the equation $x^3 - x^2 + x - 1 = 0$ the value of α is

ANSWER: sum of the roots

UNIT III

1. In the equation $X^4-X^3-7X^2+X+6=0$. Find the values of the S_4 and S_6

ANSWER: 99,795

2. If $a+b+c+d=0$, $(a^5+b^5+c^5+d^5)/5 =$

ANSWER: $(a^2+b^2+c^2+d^2)/2 \cdot (a^3+b^3+c^3+d^3)/3$

3. $X^r + 1/X^r =$

ANSWER: X_r

4. $f(x) =$

ANSWER: $X^n + P_1 X^{n-1} + \dots + P_n = 0$

5. $S_0 =$

ANSWER: n

6. $S_n, S_{n+1}, \dots, S_r$ where

ANSWER: $r > n$

7. If the degree of $\Phi(x)$ does not exceed $n-2$, prove that $\sum \alpha / \alpha_r = 0$

ANSWER: it is true

8. If $= 1$, then

ANSWER: $=$

9. If $= -1$, then

ANSWER: $= -$

10. If α be a root, the another root is

ANSWER: $1/\alpha$

UNIT IV

1) Transform the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$

ANSWER: $x^5 - 7x^3 + 12x^2 - 7x + 2$

2) The number of change of sign is

ANSWER: 8

3) The number of change of sign is 2 and the equation cannot have more than

ANSWER: two positive roots

4) Find the number of real roots of $x^7 - x^5 - x^4 - 6x^2 + 7 = 0$

ANSWER: five real roots

5) An equation $f(x) = 0$ cannot have more positive roots than there are changes of

ANSWER: sign in $f(x)$

6) If α, β, γ be the roots of $x^3 + 2x^2 + 3x + 3 = 0$, $\alpha^2/(\alpha+1)^2 + \beta^2/(\beta+1)^2 + \gamma^2/(\gamma+1)^2 =$

ANSWER: 13

7) If α, β, γ be the roots of $x^3 - x - 1 = 0$, $1 + \alpha/1 - \alpha + 1 + \beta/1 - \beta + 1 + \gamma/1 - \gamma =$

ANSWER: -7

8) The value of h in this equation $4h^3 + 3ph^2 + 2qh + r = 0$ is

ANSWER: $-p/4$

9) The equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing a roots by

ANSWER: unity

10) Solve the equation whose roots are the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ increased by 2

ANSWER: -2, -2, 2, 3

UNIT V

1. Define Row matrix:

A matrix is said to be a row matrix if it has only one row.

2. Define Column matrix:

A matrix is said to be a column matrix if it has only one column.

3. Define Square matrix:

A matrix in which the number of rows are equal to the number of columns, is said to be a Square matrix.

4. Define square matrix of order 'n'.

Thus, an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a square matrix of order 'n'.

5. Define diagonal matrix

A square matrix $B = [b_{ij}] n \times n$ is said to be a diagonal matrix if its all non diagonal elements are zero, that is a matrix $B = [b_{ij}] n \times n$ is said to be a diagonal matrix if $b_{ij} = 0$, when $i \neq j$.

6. Define scalar matrix

A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B = [b_{ij}] n \times n$ is said to be a scalar matrix if $b_{ij} = 0$, when $i \neq j$ $b_{ij} = k$, when $i = j$, for some constant k .

7. Define identity matrix

A square matrix in which elements in the diagonal are all 1 and rest are all zeroes is called

an identity matrix. In other words, the square matrix $A = [a_{ij}] n \times n$ is an identity matrix, if

$a_{ij} = 1$, when $i = j$ and $a_{ij} = 0$, when $i \neq j$.

8. Define zero matrix

A matrix is said to be zero matrix or null matrix if all its elements are zeroes. We denote zero matrix by O .

9. Define Equalent matrix

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if (a) they are of the same order, and (b) each element of A is equal to the corresponding element of B , that is, $a_{ij} = b_{ij}$ for all i and j .

10. Write the hermition matrix for $A = \begin{pmatrix} 2 & 3 - 2i \\ 2 + 3i & 4 - 3i \end{pmatrix}$

$$\text{ANSWER: } = \begin{pmatrix} -i & i \\ -i & i \end{pmatrix}$$

Classical Algebra

K3 Level Questions

UNIT I

1. Find the sum of infinity of the series

$$1 + \frac{3}{1} - \frac{3}{2} + \frac{3}{3} - \frac{3}{4} + \dots$$

2. Sum of the series to infinity

$$\frac{1}{10} - \frac{1}{10 \cdot 1} + \frac{1}{10 \cdot 1 \cdot 20} - \dots$$

3. Proof that, the co-efficient of x^n in the expansion of

$$1 + \frac{1}{1!} \frac{2x}{1} + \frac{1}{2!} \frac{2x}{2} + \dots + \frac{1}{i!} \frac{2}{i} + \dots$$

4. Sum the series

$$1 + \frac{1}{2!} \frac{3}{2} + \frac{1}{3!} \frac{3}{3} \frac{3}{2} + \frac{1}{4!} \frac{3}{4} \frac{3}{3} \frac{3}{2} + \dots$$

5. Find the co-efficient of x^n in the expansion of $e^{1-2x-3x^2}$

6. Show that if $x > 0$

$$\log x = \frac{x}{1} - \frac{1}{2} \frac{x}{x+1} + \frac{1}{3} \frac{x}{x+1} - \dots$$

7. Show that $\frac{3}{10} \log 10 = \frac{1}{2} - \frac{1}{2} \cdot \frac{3}{2^4} + \frac{1}{3} \cdot \frac{3}{2} - \dots = \log 2$

8. Using Euler's constant to prove,

$$\frac{1}{1 \cdot 2} - \frac{1}{3} + \frac{1}{4} - \dots = \log 2$$

9. Euler's constant to prove

$$\frac{1}{1 \cdot 2 \cdot 3} - \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} - \dots = 3 \log 2 - 1$$

10. Show that the co-efficient of x^n in infinite series $1 + \frac{x}{1!} - \frac{x}{2!} + \dots + \frac{x}{i!} + \dots$ by Exponential theorem.

UNIT II

1. Frame an equation with rational co-efficient one of whose roots is $\sqrt{5} - \sqrt{2}$.
2. Solve the equation $x^4 + 2x^3 - 5x^2 - 6x - 2 = 0$ given that $1 + \sqrt{-1}$ is a root of it.
3. Solve the equation $3x^3 - 4x^2 + x + 88 = 0$ which has a root $2 - \sqrt{-7}$
4. Show that the roots of the equation $x^3 + px^2 + qx + v = 0$ are in Arithmetical progression. If $2p^3 - 9pq + 27r = 0$ show that the above condition is satisfied by the equation $x^3 - 6x^2 + 13x - 10 = 0$. Hence on otherwise solve the equation.
5. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in Geometric progression solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$
6. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two. Prove that $p^3 + 8r = 4pq$.
7. Find the condition that the general biquadratic equation $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$ may have two pairs of equal roots.
8. Solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$ whose roots are in harmonic progression.
9. The roots of the equation $8x^3 - 14x^2 + 7x - 1 = 0$ are in geometrical progression. Find them.
10. Solve the equation $x^4 - 2x^3 - 21x^2 + 22x - 40 = 0$ whose roots are in arithmetical progression.

UNIT III

1. Find the sum of the fourth power of the root $x^3 - 2x^2 + x - 1 = 0$
2. Find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ where α, β, γ are roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$
3. Show that the sum of the m^{th} powers, where $m \leq n$ of the roots of the equation $x^n - 2x^{n-1} - 2x^{n-2} - \dots - 2x - 2 = 0$ is $3^m - 1$
4. Determine the value of $\phi(\alpha_1) + \phi(\alpha_2) + \dots + \phi(\alpha_n)$
5. Change the equation $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another the co-efficient of whose highest term will be unity.
6. Remove the fractional co-efficient from the equation

$$X^3 - \frac{1}{2} - \frac{1}{3} - 1 = 0$$

7. Find the sum of the fifth powers of the roots of

$$X^4 - 3x^3 + 5x^2 - 12x + 4 = 0$$

8. Find the equation whose roots are the roots of

$$X^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0 \text{ with the signs changed.}$$

9. Change the sign of the roots of the equation

$$X^7 + 4x^5 + x^3 - 2x^2 + 7x + 3 = 0$$

10. Find the roots of the equation

$$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$$

UNIT IV

1. Find the quotient and remainder when

$$2x^6 + 3x^5 - 15x^2 + 2x - 4 \text{ is divided by } x + 5$$

2. Increase by 7 the roots of the equation

$$3x^4 + 7x^3 - 15x^2 + x - 2 = 0$$

3. Find the equation each of whose roots exceeds by 2. A root of the equation $x^3 - 4x^2 + 3x - 1 = 0$

4. Find the relation between the coefficients in the equation $x^4 + px^3 + qx^2 + rx + s = 0$ in order that the coefficients of x^3 and x may be removable by the same transformation.

5. Remove the second term from the following equation:

$$X^5 + 5x^4 + 3x^3 + x^2 + x + 1 = 0$$

6. Transform the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ into one which shall want the third term.

7. Find the equation whose roots are the roots of

$$4x^5 - 2x^3 + 7x - 3 = 0 \text{ increased by 2.}$$

8. Determine completely the nature of the root of the equation $x^5 - 6x^2 - 4x + 5 = 0$

9. Find the number of real roots of the equation

$$X^3 + 18x - 6 = 0$$

10. Prove that the equation $x^4+3x-1=0$ has two real and two imaginary roots.

UNIT 5

1. Find the characteristic matrix for $A = \begin{pmatrix} 2 & 3 & 5 \\ 8 & 4 & 1 \\ 6 & 5 & 2 \end{pmatrix}$

2. Find the characteristic equation of the matrix.

$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

3. Find the Eigen vector of a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

4. write the properties of Eigen vectors

5. Find the Eigen values and Eigen vectors of a matrix

$A = \begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$

6. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$

7. Define characteristic roots and characteristic vectors.

8. Find the characteristic equation of the matrix, given below and verify that it is satisfied by the matrix.

$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

9. Diagonalise each of the following matrix.

$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$

10. Show that two similar matrices have the same characteristic roots.

Classical Algebra

K4 Level Questions

UNIT I

- Sum of the series to infinity $\frac{15}{1} - \frac{15 \cdot 1}{1 \cdot 1} + \frac{15 \cdot 1}{1 \cdot 1} - \dots$
- Find the sum to the infinity of the series $\frac{1}{1} - \frac{1}{1} + \frac{1 \cdot 5}{1 \cdot 1} - \dots$
- Sum the series:

$$1 + \frac{1}{1!} - \frac{1}{1!} + \frac{1}{1!} - \dots \text{is } e(e-1)$$

- Show that

$$\log \sqrt{12} = 1 + \left(\frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{5} + \frac{1}{5} - \dots \right)$$

- If a,b,c denote 3 consecutive integers. Show that,

$$\log_e b = \frac{1}{c} \log \frac{1}{c-1} - \frac{1}{c} \log \frac{1}{c+1} + \dots$$

UNIT II

10 Marks:

- Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one of the root is $1 - \sqrt{5}$.
- Find the equation with rational co-efficient whose root is $\sqrt{1 - \sqrt{5}}$.
- Relation between the roots and co-efficient of equation.
- Solve the equation $81x^3 - 18x^2 - 36x + 8$ whose roots are in harmonic progression.
- Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of the roots are equal in magnitude and opposite in sign.

UNIT III

- Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1 = 0$ is zero.
- If $a+b+c+d=0$ show that $\frac{c}{5} - \frac{c}{5} + \frac{c}{5} - \dots$
- If α, β, γ be the roots of the equation $x^3 - 7x + 7 = 0$, find $\frac{1}{\alpha} - \frac{1}{\beta} + \frac{1}{\gamma} - \dots$

4. Prove that the sum of the twentieth powers of the roots of the equation

$$X^4+ax+b=0, \text{ is } 50a^4b^2-4b^5.$$

5. Solve the equation $6x^5-x^4-43x^3+43x^2+x-6=0$

UNIT IV

1. Diminish the roots $x^4-5x^3+7x^2-4x+5=0$ by 2.

2. Show that the equation $x^4-3x^3+4x^2-2x+1=0$ can be transformed into a reciprocal equation by diminishing the roots by unity, hence solve the root.

3. Solve the equation $x^4+20x^3+143x^2+430x+462=0$ by removing its second term.

4. Show that $12x^7-x^4+10x^3-28=0$ has at least four imaginary roots.

5. Show that the equation $x^n-1=0$ has, when n is even, two real roots 1 and -1 and no other real root and when n is odd, the real root is 1 and no other real root.

UNIT V

1. Find the Eigen vectors of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

2. Find the Eigen vector of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

3. Find the characteristic equation of the matrix, given below and verify that it is satisfied by the matrix.

$$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

4. Diagonalise each of the following matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

5. Define matrix and write about the types of matrix.