

(FOR THE CANDIDATES ADMITTED  
DURING THE ACADEMIC YEAR  
2014 ONLY)

(NO.OF PAGES: 3)  
AIDED..... 14UMSC15  
SELF FINANCING..... 14UMAC15

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI  
END-OF-SEMESTER EXAMINATIONS : MAY - 2017  
B.Sc. – MATHEMATICS (AIDED &S.F.) MAXIMUM MARKS: 75  
VI SEMESTER TIME : 3 HOURS

**PART -III**  
**LINEAR ALGEBRA**

**SECTION – A** **(10 × 1 = 10 MARKS)**

**ANSWER THE FOLLOWING QUESTIONS. MULTIPLE CHOICE QUESTIONS.**

- If A is an  $m \times n$  matrix and  $m < n$ , then the homogeneous system of linear equation has a -----  
a) nontrivial solution      b) unique solution      c) exact solution      d) trivial solution
- A matrix which is obtained from the  $m \times m$  identity matrix by means of a single elementary row operation is called as -----  
a) Unit matrix      b) row matrix      c) Elementary matrix      d) unitary matrix
- The space V is finite dimensional if it has a -----  
a) finite basis      b) infinite basis      c) finite dimension      d) none of these
- Rank of T is the -----  
a) dimension of the range of T      b) dimension of the null space T  
c) basis      d) finite dimension
- Each  $m \times n$  matrix A is row equivalent to one and only one row reduced  
a) Unit matrix      b) echelon matrix      c) identity matrix      d) Inverse matrix

**SHORT ANSWERS.**

- Define sub field.
- Define Vector space.
- Define functional on V.
- Define linear subspace of V.
- Define nullity of T.

**SECTION – B** **(5 × 5 = 25 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

- a) Define row –reduced echelon matrix with example.

**(OR)**

- b) Show that an elementary matrix is invertible.

**(CONTD...2)**

(2)

(14UMSC15 / 14UMAC15 )

12.a) If  $A$  is an  $m \times n$  matrix over  $F$  and  $B, C$  are  $n \times p$  matrices over  $F$  then prove that

$$A(dB + C) = d(AB) + AC.$$

(OR)

b) Prove that a non-empty subset  $W$  of  $V$  is a subspace of  $V$  if and only if for each pair of vectors  $\alpha, \beta$  in  $W$  and each scalar  $C$  in  $F$  the vector  $C\alpha + \beta$  is again in  $W$ .

13.a) Let  $V$  and  $W$  be vector spaces over the field and let  $T$  be a linear transformation from  $V$  into  $W$ . If  $T$  is invertible then prove that the function  $T^{-1}$  is a linear transformation from  $W$  on to  $V$ .

(OR)

b) If  $A$  is an  $m \times n$  matrix with entries in the field  $F$ , then prove that  
row rank  $(A) =$  column rank  $(A)$ .

14.a) Let  $T$  be a linear operator on  $\mathbb{R}^2$  defined by  $T(x_1, x_2) = (x_1, 0)$  and  $[T]_\beta = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ .

Find  $[T]_\beta$  when  $\epsilon_1' = (1, 1)$ ,  $\epsilon_2' = (2, 1)$ .

(OR)

b) If  $W_1$  and  $W_2$  are subspaces of a finite dimensional vector space, then prove that  
 $W_1 = W_2$  if and only if  $W_1^\circ = W_2^\circ$ .

15.a) If  $S$  is any subset of a finite dimensional vector space  $V$ , then Prove that  $(S^\circ)^\circ$  is the subspace spanned by  $S$ .

(OR)

b) If  $f$  and  $g$  are linear functional on a vector space  $V$ , then prove that  $g$  is a scalar multiple of  $f$  if and only if the null space of  $g$  contains the null space of  $f$ , that is if and only if  
 $f(\alpha) = 0 \implies g(\alpha) = 0$ .

SECTION – C

(5 × 8 = 40 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.

16.a) If  $A, B, C$  are matrices over the field  $F$  such that the products  $BC$  and  $A(BC)$  are defined, then show that  $A(BC) = (AB)C$ .

(OR)

b) Prove that every  $m \times n$  matrix over the field  $F$  is row equivalent to row reduced matrix.

17.a) If  $W_1$  and  $W_2$  are finite dimensional subspaces of a vector space  $V$ , then prove that

$$W_1 + W_2 \text{ is finite dimensional and } \dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2).$$

(OR)

b) Let  $R$  be a non-zero row-reduced echelon matrix. Then prove that the non-zero row vectors of  $R$  form a basis for the row space of  $R$ .

(CONTD...3)

(3)

(14UMSC15 / 14UMAC15 )

18.a) Let  $V$  and  $W$  be vector spaces over the field  $F$  and  $T$  be a linear transformation from  $V$  into  $W$ . Suppose that  $V$  is finite-dimensional, then show that

$$\text{Rank}(T) + \text{nullity}(T) = \dim V.$$

(OR)

b) Prove that every  $n$ -dimensional vector space over the field  $F$  is isomorphic to the space  $F^n$ .

19.a) Let  $V$  be a finite dimensional vector space over the field  $F$ , and let  $W$  be a subspace of  $V$ . Then prove that  $\dim W + \dim W^\circ = \dim V$ .

(OR)

b) State and prove that the matrix of the  $V$  is the order basis of  $\mathcal{B}$  and  $\mathcal{B}'$ .

20.a) Let  $g, f_1, \dots, f_r$  be linear functional on a vector space  $V$  with respective null spaces  $N, N_1, \dots, N_r$ . Prove that  $g$  is a linear combination of  $f_1, \dots, f_r$  if and only if  $N$  contains the intersection  $N_1 \cap \dots \cap N_r$ .

(OR)

b) Let  $V$  and  $W$  be vector spaces over the field  $F$  and let  $t$  be a linear transformation from  $V$  into  $W$ . The null space of  $T^t$  is the annihilator of the range of  $T$ . If  $V$  and  $W$  are finite dimensional then prove that (i)  $\text{rank}(T^t) = \text{rank}(T)$

(ii) the range of  $T^t$  is the annihilator of the null space of  $T$ .

A6/.

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DURING THE ACADEMIC YEARS AIDED.....

14 UMSC 16 / 13 UMSC 15

2013 AND 2014 ONLY )

SELF FINANCING..

14 UMAC 16 / 13 UMAC 15

N.G.M.COLLEGE (AUTONOMOUS) : POLLACHI

END-OF-SEMESTER EXAMINATIONS : MAY - 2017

B.Sc.- MATHEMATICS

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VI SEMESTER

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## PART -III

## REAL ANALYSIS II

SECTION – A

(10 × 1 = 10 MARKS)

ANSWER THE FOLLOWING QUESTIONS. MULTIPLE CHOICE QUESTIONS.

- Let  $f$  and  $g$  are defined on  $(a,b)$  and differentiable at  $c$ , then  $(f \pm g)'(c) =$ \_\_\_\_\_
  - $f'(c) + g'(c)$
  - $f'(c) - g'(c)$
  - $f'(c) \pm g'(c)$
  - $f'(c) = g'(c)$
- If  $f$  is monotonic  $[a,b]$  then the set of discontinuity of  $f$  is\_\_\_\_\_
  - finite
  - Infinite
  - countable
  - un countable
- For every  $\epsilon > 0$  there exist a partition  $P_\epsilon$  of  $[a,b]$  such that  $|s(p,f,\alpha) - A| < \epsilon$  whenever  $P \supseteq P_\epsilon$  and for any  $t_k \in [x_{k-1}, x_k]$  where  $A$  is denoted by
  - $\int_a^b f d\alpha$
  - $\int_a^b f d$
  - $\int_a^b f \alpha$
  - None of the above
- $L(p,f,\alpha) \leq S(p,f,\alpha) \leq U(p,f,\alpha)$ , the inequality is not true when  $\alpha$  is\_\_\_\_\_ on  $[a,b]$ .
  - increasing
  - not increasing
  - decreasing
  - not decreasing
- If  $f$  is continuous on  $[a,b]$ , there exist a point  $x_0$  in  $[a,b]$  such that  $f(x_0) = C$  then it is called as\_\_\_\_\_
  - First mean value theorem
  - Second mean value theorem
  - Intermediate value theorem
  - None

(CONTD...2)

(2) (13 UMSC 15 / 13 UMAC 15/14UMSC16 / 14UMAC16)

**SHORT ANSWERS.**

6. State the Mean Value Theorem.
7. Define bounded variation.
8. Define Step Function.
9. Let  $\alpha$  increasing on  $[a,b]$  then for any two partitions  $p_1$  and  $p_2$  we have  $L(p_1, f, \alpha) = U(p_2, f, \alpha)$ . Say True or False.
10. State Second Mean Value Theorem for Riemann stieltjes integrals.

**SECTION – B**

(5 × 5 = 25 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

- 11.a) If  $f$  is differentiable at  $c$ , then prove that  $f$  is continuous at  $c$ .

**(OR)**

- b) State and prove the Chain Rule Theorem for Differentiation.

- 12.a) If  $f$  is monotonic on  $[a,b]$ , then prove that  $f$  is bounded variation on  $[a,b]$ .

**(OR)**

- b) Let  $f$  be continuous on  $[a,b]$ , if  $f$  is of bounded variation on  $[a,b]$  then

prove that  $f$  can be expressed as the difference of two increasing functions.

- 13.a) If  $f \in R(\alpha)$  and  $f \in R(\beta)$  on  $[a,b]$  then prove that  $f \in R(c_1\alpha + c_2\beta)$  on  $[a,b]$  ( for any two constant

$$c_1 \text{ and } c_2) \text{ and } \int_a^b f d(c_1\alpha + c_2\beta) = c_1 \int_a^b f d\alpha + c_2 \int_a^b f d\beta$$

**(OR)**

- b) If  $f \in R(\alpha)$  and if  $g \in R(\alpha)$  on  $[a,b]$ , then prove that  $c_1f + c_2g \in R(\alpha)$  on  $[a,b]$  ( for any two constant

$c_1$  and  $c_2$ ) and

$$\int_a^b (c_1f + c_2g) d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha.$$

- 14.a) Let  $\alpha$  is increasing on  $[a,b]$  then prove that  $\underline{I}(f, \alpha) \leq \bar{I}(f, \alpha)$ .

**(OR)**

- b) Let  $\alpha$  is increasing function on  $[a,b]$ ,  $f \in R(\alpha)$  on  $[a,b]$ , then prove that  $|f| \in R(\alpha)$  on  $[a,b]$ , and

prove the inequality

$$\left| \int_a^b f(x) d\alpha(x) \right| \leq \int_a^b |f(x)| d\alpha(x).$$

**(CONTD...3)**

(3) (13 UMSC 15 / 13 UMAC 15/14UMSC16 / 14UMAC16)

15.a) State and prove Second Mean Value theorem for Riemann Stieltjes Integrals.

(OR)

b) State and prove First Mean Value theorem for Riemann stieltjes integrals.

**SECTION – C** (5 × 8 = 40 MARKS)

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

16.a) i) State and prove Rolle's theorem.

ii) Prove that if  $f$  has a derivative (finite or infinite) at each point of an open interval  $(a,b)$  and  $f$  is continuous at both end points  $a$  and  $b$  then there is a point  $c$  in  $(a,b)$  such that  $f(b)-f(a)=f'(c)(b-a)$ .

(OR)

b) If  $f$  and  $g$  be two functions, each having a derivative (finite or infinite) at each point of an open interval  $(a,b)$  and each continuous at the endpoints  $a$  and  $b$ . Assume also that there is no interior point  $x$  at which both  $f'(x)$  and  $g'(x)$  are infinite. Then prove that for some interior point  $c$  we have

$$f'(c)[g(b)-g(a)]=g'(c)[f(b)-f(a)].$$

17.a) If  $f$  and  $g$  are each of bounded variation on  $[a,b]$ . Then prove that

$$v_{f \pm g} \leq v_f + v_g \text{ and } v_{fg} \leq Av_f + Bv_g$$

Where  $A = \sup \{|g(x)| : x \in [a,b]\}$ ,

$B = \sup \{|f(x)| : x \in [a,b]\}$ .

(OR)

b) Let  $f$  be of bounded variation on  $[a,b]$ , if  $x \in (a,b)$ ,  $v(x) = v_f(a,x)$  and  $v(a) = 0$ .

Then prove that every point of continuity of  $f$  is also a point of continuity of  $v$  and the converse is also true.

18.a) If  $f \in R(\alpha)$  on  $[a,b]$  then prove that  $\alpha \in R(f)$  on  $[a,b]$  and  $\int_a^b f(x) d\alpha(x) + \int_a^b \alpha(x) df(x) = f(b)\alpha(b) - f(a)\alpha(a)$ .

(OR)

b) State and prove Euler's summation formula.

(CONTD...4)

(4) (13 UMSC 15 / 13 UMAC 15/14UMSC16 / 14UMAC16)

19.a) *Let  $\alpha$  increasing on  $[a, b]$ .* Then prove that the following three statements are equivalent.

- i)  $f \in R(\alpha)$  on  $[a, b]$
- ii)  $f$  satisfies Riemann's condition with respect to  $\alpha$  on  $[a, b]$ .
- iii)  $\underline{I}(f, \alpha) = \bar{I}(f, \alpha)$ .

(OR)

b) Prove that if  $\alpha$  is of bounded variation on  $[a, b]$  assume that  $f \in R(\alpha)$  on  $[a, b]$ . Then prove that  $f \in R[\alpha]$  on every subinterval  $[c, d]$  of  $[a, b]$ .

20.a) If  $g$  has a continuous derivative  $g^1$  on a interval  $[c, d]$ . Let  $f$  be continuous on  $g[c, d]$  and define  $F$  by equation  $F(x) = \int_{g(c)}^x f(t) dt$ . if  $x \in g[c, d]$ , then prove that for each  $x$  in  $[c, d]$  the integral  $\int_c^x f[g(t)] g^1(t) dt$  exists and has the value  $F[g(x)]$ .

In particular prove that  $\int_{g(c)}^{g(d)} f(x) dx = \int_c^d f[g(t)] g^1(t) dt$ .

(OR)

b) If  $f \in R$  and  $g \in R$  on  $[a, b]$ . Let  $F(x) = \int_a^x f(t) dt$ ,  $G(x) = \int_a^x g(t) dt$  if  $x \in [a, b]$ . Then  $F$  &  $G$  are continuous functions of bounded variation on  $[a, b]$ . Also  $f \in R(G)$  and  $g \in R(F)$  on  $[a, b]$  and prove that,

$$\int_a^b f(x) g(x) dx = \int_a^b f(x) dG(x) = \int_a^b g(x) dF(x).$$

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A6/.

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14 UMSC 17

S.F. ....

14 UMAC 17

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

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MAXIMUM MARKS: 75

VI SEMESTER

TIME: 3 HOURS

### PART – III

### COMPLEX ANALYSIS - II

### SECTION – A

(10 X 1 = 10 MARKS)

ANSWER ALL QUESTIONS. MULTIPLE CHOICE QUESTIONS.

- The direction of the tangent to the curve  $r : z(t)$  is given by  
a)  $\arg z(t)$       b)  $\arg z'(t)$       c)  $\arg z''(t)$       d)  $\arg z'''(t)$
- The value of the linear fractional transformation  $w = s(z) = \frac{az+b}{cz+d}$  at  $z = \infty$  is  
a)  $a/c$       b)  $c/a$       c)  $d/a$       d)  $a/d$
- $\int_r |dz|$  where  $r$  is the circle given by  $z(t) = a + \rho e^{it}$ ,  $0 \leq t \leq 2\pi$  is  
a)  $2\pi i$       b)  $2\pi \rho$       c)  $2\pi a$       d)  $2\pi$
- If  $r$  lies inside a circle, then for all points  $a$  outside the same circle  $n(r, a)$  is  
a) 1      b)  $\infty$       c) 0      d) constant
- If  $\lim_{z \rightarrow a} f(z) = \infty$ ,  $z = a$  is a  
a) Pole      b) isolated singularity      c) essential singularity      d) zero
- Define analytic function.
- Define Cross ratio.
- What is the complex line integral of  $f(z)$  over the arc  $r$  with the equation  $z = z(t)$ ,  $a \leq t \leq b$ .
- Define index of the point  $a$  with respect to the curve  $r$ .
- What is a meromorphic function?

(CONTD.....2)

**(2)**  
**SECTION – B**

**(14 UMSC 17/14 UMAC 17)**  
**(5 X 5 = 25 MARKS)**

**ANSWER EITHER ‘a’ OR ‘b’ IN EACH OF THE FOLLOWING QUESTIONS.**

11. a) Prove that the mapping  $w = f(z)$  is conformal at all points with  $f'(z) \neq 0$ .  
(OR)  
b) Obtain the complex form of the Cauchy – Riemann equations.
12. a) Prove that linear transformations form a group.  
(OR)  
b) if  $Z_1, Z_2, Z_3, Z_4$  are distinct points in the extended plane and  $T$  is any linear transformation then prove that  $(TZ_1, TZ_2, TZ_3, TZ_4) = (Z_1, Z_2, Z_3, Z_4)$
13. a) Evaluate  $\int_C \frac{dz}{z-a}$  where  $C$  is the circle  $z = a + f e^{it}$ ,  $0 \leq t \leq 2\pi$ .  
(OR)  
b) Evaluate  $\int_r x dz$  where  $r$  is the directed line segment from 0 to  $1 + i$ .
14. a) Compute  $\int_{|z|=1} \frac{e^z}{z} dz$   
(OR)  
b) State and prove Liouville's theorem.
15. a) Show that the functions  $w = e^z$  and  $w = \sin z$  have essential singularities at  $\infty$ .  
(OR)  
b) Prove that a non – constant analytic function maps open sets onto open sets.

**SECTION – C**

**(5 X 8 = 40 MARKS)**

**ANSWER EITHER ‘a’ OR ‘b’ IN EACH OF THE FOLLOWING QUESTIONS.**

16. a) Prove that an analytic function in a region  $\Omega$  whose derivative vanishes identically must reduce to a constant. Prove that the same is true if either the real part, the imaginary part is constant.  
(OR)  
b) Discuss the analyticity of  $\log z$ .

**(CONTD.....3)**

(3)

(14 UMSC 17/14 UMAC 17)

17. a) Prove that a linear transformation carries circles into circles.  
(OR)
- b) If  $T_1 Z = \frac{Z+2}{Z+3}$ ,  $T_2 Z = \frac{Z}{Z+1}$  find  $T_1 T_2 z$ ,  $T_2 T_1 Z$  and  $T_1^{-1} T_2 Z$ .
18. a) Prove that the line integral  $\int_r p dx + q dy$  defined in  $\Omega$  depends only on the end points of  $r$  if and only if there exists a function  $U(x, y)$  in  $\Omega$  with the partial derivative  $\frac{\partial U}{\partial x} = p$   $\frac{\partial U}{\partial y} = q$ .  
(OR)
- b) State and prove Cauchy's theorem.
19. a) If the Piecewise differentiable closed curve  $r$  does not pass through the point  $a$ , then prove that the value of the integral  $\int_r \frac{dz}{z-a}$  is a multiple of  $2\pi i$ .  
(OR)
- b) Suppose that  $\phi(\ )$  is continuous of the arc  $r$ . Then prove that the function  $F_n(z) = \int_r \frac{\phi(\ ) d}{(\ -z)^n}$  is analytic in each of the regions determined by  $r$  and its derivative is  $F_n'(z) = n F_{n+1}(z)$ .
20. a) State and prove Taylor's theorem.  
(OR)
- b) Define Essential singularity and characterize it.



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14 UMSC 18

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14 UMAC18

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PART – III

DISCRETE MATHEMATICS

SECTION – A

(10 X 1 = 10 MARKS)

ANSWER ALL QUESTIONS.

1. The Fibonacci sequence is defined by the relation\_\_\_\_\_

(a)  $F_n = F_{n-1} + F_{n-2}$       (b)  $F_{n+1} = F_{n-1} + F_{n-2}$       (c)  $F_n^2 = F_{n-1} + F_{n-2}$       (d)  $F_n = F_{n-2} + F_{n-4}$

2. The Generating Function of a sequence  $S_0, S_1, S_2, \dots$  is \_\_\_\_\_

(a)  $S_0 + S_1Z + S_2Z^2$       (b)  $S_0 - S_1Z - S_2Z^2$       (c)  $S_0 - S_1Z + S_2Z^2$       (d)  $1/S_0 + 1/S_1Z + 1/S_2Z^2$

3. If P: It is raining and Q: 7 is a prime number then It is raining or 7 is a prime number can be denoted as \_\_\_\_\_

(a)  $P \vee Q$       (b)  $P \wedge Q$       (c)  $7P$       (d)  $P \vee 7Q$

4. The Dual of  $P \wedge (7Q \vee R)$  is\_\_\_\_

(a)  $P \vee (7Q \vee R)$       (b)  $P \vee (7Q \wedge R)$       (c)  $P \wedge (7Q \wedge R)$       (d)  $7P \wedge (7Q \vee R)$

5. A Relation R is said to be an Equivalence relation if R is \_\_\_\_\_

- (a) Reflexive , Symmetric and Transitive      (b) irreflexive , Symmetric and Transitive  
(c) reflexive , anti-Symmetric and anti-Transitive      (d) Reflexive , anti- Symmetric and Transitive

6. Write any one of the Principles of Strong Mathematical Induction.

7. When a total function f over N is primitive recursive?

8. Define Logic.

9. Define Contradiction.

10. Define Poset.

(CONTD.....2)

**PART-B****(5×5 =25 MARKS)****CHOOSING EITHER (a) OR (b).**11.(a) Calculate  $F_4$  of the Fibonacci numbers using (i) recursion (ii) Iteration**(OR)**(b) Prove by Mathematical induction that  $Z^n > n \forall n \in \mathbb{N}$ .

12.(a) Find the Generating function of the Fibonacci sequence.

**(OR)**(b) Find  $T(2T)$  where  $T(n)$  denotes the worst time for binary search of a file with “n” records.

13.(a) Explain Well-formed formula.

**(OR)**(b) Construct the truth table for  $(7P \vee Q) \wedge (7Q \vee P)$ .14.(a) Show that  $Q \vee (P \wedge 7Q) \vee (7P \wedge 7Q)$  is a Tautology.**(OR)**(b) Show that  $(P \rightarrow Q) \wedge (R \rightarrow Q)$  and  $(P \vee R) \rightarrow Q$  are equivalent formulae.

15.(a) Prove that every chain is a Lattice .

**(OR)**(b) Show that a Lattice  $L$  is a modular iff  $\forall x, y, z \in L$  then  $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$ **PART-C****(5×8 =40 MARKS)****CHOOSING EITHER (A) OR (B).**16.(a) Show that  $(a^n - b^n)$  is divisible by  $(a-b) \forall n \in \mathbb{N}$ .**(OR)**(b) Find  $f(x)$  if  $f(n) = 7f(n-1) - 10f(n-2)$  given that  $f(0) = 4$  and  $f(1) = 17$ 17.(a) Show that  $T(k) - 7T(k-1) + 10T(k-2) = 6 + 8k$  with  $T(0) = 1$  and  $T(1) = 2$ **(OR)**(b) Show that  $f(x, y) = x^y$  is primitive recursive.

18.(a) Explain procedure to draw the parsing tree.

**(OR)**

(b) Construct the truth table for

(i)  $P \rightarrow (Q \rightarrow R)$  (ii)  $(Q \vee R) \rightarrow (P \wedge 7R)$ 19.(a) Establish that  $7(P \wedge Q) \rightarrow (7P \vee (7P \vee Q)) \Rightarrow (7P \vee Q)$ .**(OR)**(b) Find the PDNF and PCNF of  $(P \wedge Q) \vee (7P \wedge Q) \vee (Q \wedge R)$ 20.(a) Show that a Lattice  $L$  is distributive iff  $\forall a, b, c$  in  $L$  then  $(a \vee b) \wedge c \leq a \vee (b \wedge c)$ **(OR)**(b) Prove that  $(L \times M, \wedge, \vee)$  is a Lattice.

(FOR THE CANDIDATES ADMITTED  
DURING THE ACADEMIC YEAR  
2014 ONLY)

(NO. OF PAGES: 2)

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14 UMSC 19

S.F. ....

14 UMAC 19

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

B.Sc. - MATHEMATICS

MAXIMUM MARKS: 75

VI SEMESTER

TIME: 3 HOURS

**PART – III**

**OBJECT ORIENTED PROGRAMMING WITH C++**

**SECTION – A**

**(10 X 1 = 10 MARKS)**

**ANSWER ALL QUESTIONS.**

- \_\_\_\_\_ refers to the act of representing essential features without including the background details.  
a) Abstraction    b) Encapsulation    c) Inheritance    d) Association
- \_\_\_\_\_ is a function that is expanded in line when it is invoked  
a) main( )    b) Inline    c) Friend    d) Const member
- A data member of a class can be declared as a \_\_\_\_\_ and is normally used to maintain values common to the entire class.  
a) extern    b) static    c) Friend    d) protected
- A constructor that accepts no parameters is called \_\_\_\_\_.  
a) Dynamic Constructor    b) Default Constructor  
c) Copy Constructor    d) Parameterized Constructor
- By default, members of the class are \_\_\_\_\_.  
a) public    b) protected    c) private    d) static
- What is member variable?
- Define function
- Define Object.
- What is Function over loading?
- Define inheritance.

**(CONTD.....2)**

(2)

(14 UMSC 19/ 14 UMAC 19)

**SECTION B**

[5 X 5 = 25 MARKS]

**ANSWER EITHER A OR B IN EACH OF THE FOLLOWING QUESTIONS.**

11. a. (i) List out the advantages of Object Oriented Programming method. (3 marks)  
(ii) Write short notes on Structure C++ program (2 marks)  
[OR]  
b. Explain While loop in C++ with examples.
12. a. Give a brief account on Inline function.  
[OR]  
b. Write short note on argument passed by reference with suitable examples.
13. a. Explain specifying a class.  
[OR]  
b. How to access class member?
14. a. Explain operator overloading with example.  
[OR]  
b. Write short notes on pointer with example.
15. a. (i) What is a virtual base class? When does it make a class virtual? (3 marks)  
(ii) Define abstract class. Give an example. (2 marks)  
[OR]  
b. Write a short note on Nesting of class

**SECTION C**

[5 X 8 = 40 MARKS]

**ANSWER EITHER A OR B IN EACH OF THE FOLLOWING QUESTIONS.**

16. a. Explain Conditional structures in C++ in detail with examples.  
[OR]  
b. What is an Operator? Explain different types of operators in detail with example.
17. a. What is Overloading? Write a program to illustrate function overloading.  
[OR]  
b. Discuss multiple constructors in a class with example.
18. a. Explain briefly static data member.  
[OR]  
b. Describe briefly friendly function.
19. a. Briefly explain the overloading Unary Operators.  
[OR]  
b. Describe Virtual function with example. State the rules for using virtual function.
20. a. Explain briefly Multiple Inheritance.  
[OR]  
b. Explain briefly Hierarchical Inheritance.

(FOR THE CANDIDATES ADMITTED

(NO. OF PAGES: 3)

DURING THE ACADEMIC YEARS

AIDED .....

16 UMS 2A2 / 15 UMS2A2

2016 ONLY)

S.F. ....

16 UMA 2A2/ 15 UMA2A2

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

B.Sc. - MATHEMATICS (AIDED & S.F.)

MAXIMUM MARKS: 75

II SEMESTER

TIME: 3 HOURS

PART – III

## MATHEMATICAL STATISTICS-II

### SECTION – A

(10 X 1 = 10 MARKS)

ANSWER ALL QUESTIONS. CHOOSE THE BEST ANSWER.

1. A two-dimensional random variable is said to be discrete, if it takes atmost a \_\_\_\_\_ number of points in  $R^2$ .  
a) Finite                      b) Infinite                      c) Countable                      d) Uncountable
2. If  $X$  and  $Y$  are independent variables, then  $Cov(X, Y)$  is  
a) 1                      b) 2                      c) 0                      d)  $\infty$
3. A sample is said to be large, if  
a)  $n > 30$                       b)  $n < 30$                       c)  $n > 100$                       d)  $n < 100$
4. \_\_\_\_\_ is known as mean-square contingency.  
a)  $\phi^2 = \frac{\chi^2}{N}$                       b)  $\phi^2 = \frac{\varphi^2}{N}$                       c)  $\phi^2 = \frac{\psi^2}{N}$                       d)  $\phi^2 = \frac{\delta}{N}$
5. \_\_\_\_\_ is used to test the significance of observed partial correlation coefficient.  
a) t-distribution                      b) F-distribution                      c)  $\chi^2$ -distribution                      d) Normal distribution

### II. SHORT ANSWER QUESTIONS.

6. Define conditional probability function.
7. Explain Regression coefficients.
8. Explain One-tailed test.
9. Write the formula for  $\chi^2$ -test.
10. Write any two assumption for students t-distribution.

(CONTD.....2)

**(2) (16 UMS2A2/ 16 UMA2A2/ 15 UMS2A2/ 15 UMA2A2)**  
**SECTION – B** **(5 X 5 = 25 MARKS)**

ANSWER EITHER 'a' OR 'b' IN EACH OF THE FOLLOWING QUESTIONS.

11.a) Derive the Marginal probability function.

(OR)

b) Let  $(x, y) = 8xy$ ,  $0 < x < y < 1$ ;  $f(x, y) = 0$  elsewhere.

Find (i)  $E(Y/X = x)$

(ii)  $E(XY/X = x)$

12. a) The joint probability distribution of X and Y is given below.

| $X \backslash Y$ | -1    | +1    |
|------------------|-------|-------|
| 0                | $1/8$ | $3/8$ |
| 1                | $2/8$ | $2/8$ |

Find the correlation coefficient between X and Y.

(OR)

b) Derive the angle between the two lines of Regression.

13.a) A random sample of 500 pineapples were taken from a large consignment and 65 were found to be bad. Show that the S.E. of the proportion of bad ones in a sample of this size is 0.015 and deduce that the percentage of bad pineapples in the consignment almost certainly between 8.5 and 17.5.

(OR)

b) The guaranteed average life of a certain type of electric light bulbs is 1,000 hours with a standard deviation of 125 hours. It is decided to sample the output so as to ensure that 90 percent of the bulbs do not fall short of the guaranteed average by more than 2.5 percent. What must be the minimum size of the sample?

14.a) Test the hypothesis that  $\sigma = 10$ , given that  $s = 15$  for a random sample size 50 from a normal population.

(OR)

b) Explain the applications of chi-square distribution.

15.a) The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful?

(OR)

b) Test the claim that the resistance of electric wire can be reduced by at least 0.05 ohm by alloying, 25 values obtained for each alloyed wire and standard wire produced the following results.

|               | Mean      | Standard Deviation |
|---------------|-----------|--------------------|
| Alloyed wire  | 0.083 ohm | 0.003 ohm          |
| Standard wire | 0.136 ohm | 0.002 ohm          |

Test at 5% level whether or not the claim is substantiated.

**SECTION – C**

**(5 X 8 = 40 MARKS)**

ANSWER EITHER 'a' OR 'b' IN EACH OF THE FOLLOWING QUESTIONS.

16.a) Two ideal dice are thrown. Let  $X_1$  be the score on the first die and  $X_2$  the score on the second die. Let Y denotes the maximum of  $X_1$  and  $X_2$ . i.e,  $Y = \max(X_1, X_2)$ .

(i) Write down the joint distribution of Y and  $X_1$ .

(ii) Find the mean and variance of Y and covariance of  $(Y, X_1)$ .

(OR)

**(CONTD.....3)**

- b) Two random variables  $X$  and  $Y$  have the following joint probability density function.

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0; & \text{Otherwise} \end{cases}$$

- Find (i) Marginal probability density function. (ii) Conditional density function  
(iii)  $\text{var}(X)$  and  $\text{var}(Y)$  (iv) Covariance between  $X$  and  $Y$ .

- 17.a) Prove that the Correlation coefficient is independent of change of origin and scale.

(OR)

- b) Ten competitors in a musical test were ranked by the judges A, B and C in the following order.

Ranks by A : 1 6 5 10 3 2 4 9 7 8

Ranks by B : 3 5 8 4 7 10 2 1 6 9

Ranks by C : 6 4 9 8 1 2 3 10 5 7

Using rank correlation method, discuss which pair of judges has the nearest approach to common likings in music.

- 18.a) A die is thrown 9,000 times and a throw of 3 or 4 is observed 3,240 times. Show that the die cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.

(OR)

- b) The standard deviation of a population is 2.70cms. Find the probability that in a random sample of size 66 (i) the sample mean will differ from the population mean by 0.75cm. or more, and (ii) the sample mean will exceed the population mean by 0.75cm. or more (given the value of the standard normal probability integral from 0 to 2.25 is 0.4877).

- 19.a) When the first proof of 392 pages of a book of 1200 pages were read, the distribution of printing mistakes were found to be as follows.

No. of mistakes in a page ( $x$ ) : 0 1 2 3 4 5 6

No. of pages ( $f$ ) : 275 72 30 7 5 2 1

Fit a Poisson distribution to the above data and test the of goodness of fit.

(OR)

- b) For the  $2 \times 2$  contingency table,

, Prove 

|   |   |
|---|---|
| a | b |
| c | d |

 Prove that chi-square test of independence gives

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}, \text{ where } N = a + b + c + d.$$

- 20.a) Below are given the gain in weights (in kgs.) of pigs fed on two diets A and B.

**Gain in weight**

Diet A: 25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25

Diet B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22

Test, if the two diets differ significantly as regards their effect on increase in weight.

(OR)

- b) Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show the sample standard deviations of their weight distributions are normal. Test the hypothesis that the true variances are equal, against the alternative that they are not, at the 10% level [Assume that  $P(F_{10,8} \geq 3.35) = 0.05$  and  $P(F_{10,8} \geq 3.07) = 0.05$ ].

(FOR THE CANDIDATES ADMITTED  
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2015 ONLY)

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15 UMS 407

S.F. ....

15 UMA 407

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

B.Sc. - MATHEMATICS (AIDED S.F.)

MAXIMUM MARKS: 75

IV SEMESTER

TIME: 3 HOURS

PART – III

STATICS

SECTION – A

(10 X 1 = 10 MARKS)

ANSWER ALL QUESTIONS.

1. If the forces P and Q are at right angles to each other, then  $\cos \alpha$  is \_\_\_\_\_.  
a) 1                      b) 0                      c) -1                      d) 1
2. The algebraic sum of the moments of any number of coplanar forces about any point on the line of action of their resultant is \_\_\_\_\_.  
a) one                      b) equal                      c) zero                      d) none of these
3. Two equal and unlike parallel forces not acting at the same point are said to constitute a \_\_\_\_\_.  
a) arm                      b) couple                      c) moment                      d) none of these
4. When a rod is resting on a smooth peg, the reaction of the peg on the rod is \_\_\_\_\_ to the rod.  
a) opposite                      b) parallel                      c) perpendicular                      d) none of these
5. The angle of repose of a rough inclined plane is equal to the \_\_\_\_\_.  
a) angle of friction                      b) cone of friction                      c) friction                      d) none of these
6. State Lami's theorem.
7. What is the condition of equilibrium of three coplanar parallel forces?
8. What is the arm of the couple?
9. When are three forces acting on a rigid body coplanar?
10. What is friction?

(CONTD.....2)

(2)

(15 UMS 407/15 UMA407)

**SECTION – B**

**(5 X 5 =25 MARKS)**

**ANSWER EITHER (A) OR (B) IN EACH OF THE FOLLOWING QUESTIONS.**

11. (a) Two forces act on a particle. If the sum and difference of the forces are at right angles to each other, show that the forces are of equal magnitude.

**(OR)**

- (b) OA, OB, OC are the lines of action of two forces P and Q and their resultant R respectively. Any transversal meets the lines in L, M and N respectively. Prove that

$$\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}.$$

12. (a) Three like parallel forces, acting at the vertices of a triangle, have magnitudes proportional to the opposite sides. Show that their resultant passes through the incentre of the triangle.

**(OR)**

- (b) At what height from the base of a pillar must the end of a rope of a given length be fixed so that a man standing on the ground and pulling at its other end with a given force may have the greatest tendency to make the pillar over turn?

13. (a) Prove that two couples in the same plane whose moments are equal and of the same sign are equivalent to one another.

**(OR)**

- (b) Prove that a couple and a single force acting on a body can not be in equilibrium but they are equivalent to the single force acting at some other point parallel to its original direction.

14. (a) A uniform solid hemisphere of weight W rests with its curved surface on a smooth horizontal plane. A weight w is suspended from a point on the rim of the hemisphere.

If the plane base of the rim is inclined to the horizontal at an angle  $\theta$ , prove that  $\tan \theta = \frac{8w}{3W}$ .

**(OR)**

- (b) Forces acting along the sides of a quadrilateral ABCD are in equilibrium. If the diagonals AC, BD meet at O, show that the force acting along any side is inversely proportional to the distance of O from that side.

15. (a) Prove that the angle of repose of a rough inclined plane is equal to the angle of friction.

**(OR)**

- (b) In a solid hemisphere of radius a, the density varies inversely as the distance from the centre. Find the position of the centre of mass.

**(CONTD.....3)**

(3)  
**SECTION – C**

(15 UMS 407/15 UMA407)  
(5 X 8 =40 MARKS)

**ANSWER EITHER (A) OR (B) IN EACH OF THE FOLLOWING QUESTIONS.**

16. (a) The resultant of two forces P, Q acting at a certain angle is X and that of P, R acting at the same angle is also X. The resultant of Q, R again acting at the same angle is Y.

Prove that  $P = (X^2 + QR)^{\frac{1}{2}} = \frac{QR(Q+R)}{Q^2 + R^2 - Y^2}$ . Prove also that, if  $P+Q+R = 0$ , then  $Y = X$ .

**(OR)**

- (b) P is a point in the plane of the triangle ABC and I is the incentre. Show that the resultant of the forces represented by  $PA \sin A$ ,  $PB \sin B$ ,  $PC \sin C$  is

$$4PI \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

17. (a) Prove that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about that point.

**(OR)**

- (b) The resultant of three forces P, Q, R acting along the sides BC, CA, AB of a triangle ABC passes through the orthocenter. Show that the triangle must be obtuse angled. If

$$\angle A = 120^\circ \text{ and } B = C, \text{ then show that } Q + R = P\sqrt{3}.$$

18. (a) Prove that the resultant of any number of couples in the same plane on a rigid body is a single couple whose moment is equal to the algebraic sum of the moments of the several couples.

**(OR)**

- (b) ABC is an equilateral triangle of side a; D, E, F divide the sides BC, CA, AB respectively in the ratio 2:1. Three forces each equal to P act at D, E, F perpendicularly to the sides and outward from the triangle. Prove that they are equivalent to a couple of moment  $\frac{1}{2} Pa$ .

19. (a) If D is any point on the base BC of a triangle ABC such that  $\frac{BD}{DC} = \frac{m}{n}$  and

$\angle ADC = \theta, \angle BAD = \alpha, \angle DAC = \beta$ , prove the following:

$$(m+n) \cot \theta = m \cot \alpha - n \cot \beta \text{ and } (m+n) \cot \theta = n \cot B - m \cot C.$$

**(OR)**

**(CONTD.....4)**

- (b) Two equal uniform rods, each of weight  $W$  and length  $a$ , are freely joined at one extremity, and rest symmetrically in a vertical plane in contact with a smooth horizontal cylinder of radius  $b$ , the axis of the cylinder being at right angles to the plane of the rods. Their other extremities are connected by an inextensible string of length  $2c$ . Show that its tension is  $w \left( \frac{ab}{c^2} - \frac{c}{2\sqrt{a^2 - c^2}} \right)$ .

20. (a) A uniform sphere is held in equilibrium on a rough inclined plane of angle  $\alpha$

by a force of magnitude  $\frac{W}{2} \sin \alpha$  applied tangentially to its circumference, where  $W$  is the weight of the sphere. Prove that the force must act parallel to the plane and that the coefficient of friction must be not less than  $\frac{1}{2} \tan \alpha$ .

(OR)

- (b) From a uniform solid cylinder, a cone whose base coincides with the base of the cylinder is scooped out so that the C.G. of the remaining solid coincides with the vertex of the cone. What is the height of the cone?

(FOR THE CANDIDATES ADMITTED  
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2016 ONLY)

(NO. OF PAGES: 3)

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16 UMS 203

S.F. ....

16 UMA 203

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

B.Sc. - MATHEMATICS (AIDED S.F.)

MAXIMUM MARKS: 75

II SEMESTER

TIME: 3 HOURS

**PART – III**

**TRIGONOMETRY, VECTOR CALCULUS AND FOURIER SERIES**

**SECTION – A**

**(10 X 1 = 10 MARKS)**

**ANSWER ALL QUESTIONS.**

1.  $x^n + \frac{1}{x^n} = \text{-----}$

- a)  $2i \cos n\theta$       b)  $2 \cos \theta$       c)  $2 \cos n\theta$       d)  $2 \cos 2\theta$

2.  $\sinh^{-1} x = \text{-----}$

- a)  $\log_e (x + \sqrt{x^2 + 1})$       b)  $\log_e (x - \sqrt{x^2 - 1})$   
c)  $\log_e (x - \sqrt{x^2 + 1})$       d) none of the above

3.  $\int_0^\pi \cos nx \, dx = \text{-----}$  if m is integer

- a) 1      b) 0      c) 2      d)  $\infty$

4.  $\text{Curl} (\phi \nabla \phi) = \text{-----}$

- a) 0      b) 1      c) -1      d) none of these

5.  $\text{div} (\text{grad}) \left[ \frac{1}{r} \right] = \text{-----}$

- a) 1      b) -1      c) 0      d) r

6. Expand  $\sin \theta$  in a series of ascending powers of  $\theta$ .

7. Write Euler's formula of Hyperbolic functions.

8. Define an odd function.

9. Define solenoidal field.

10. Find the unit normal to the surface  $xy^3z^2 = 4$  at the point  $(-1, -1, 2)$ .

**(CONTD.....2)**

**(2)**  
**SECTION – B**

**(16 UMS 203/ 16 UMA 203)**  
**(5 X 5 = 25 MARKS)**

**ANSWER EITHER ‘a’ OR ‘b’ IN EACH OF THE FOLLOWING QUESTIONS.**

11. a) Express  $\cos 8\theta$  in terms of  $\sin \theta$ .  
(OR)  
b) Solve approximately in radians  $\sin \left[ \frac{\pi}{3} + x \right] = 0.87$
12. a) If  $\tan (x + iy) = u + iv$ , prove that  $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$   
(OR)  
b) Prove:  $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$ .
13. a) Find a sine series for  $f(x) = c$  in the range 0 to  $\pi$ .  
(OR)  
b) Express  $f(x) = \frac{1}{2}(\pi - x)$  as a Fourier series with period  $2\pi$ , to be valid in the interval 0 to  $2\pi$ .
14. a) Prove that  $(3x + 2y + 4z)\vec{i} + (2x + 5y + 4z)\vec{j} + (4x + 4y - 8z)\vec{k}$  is both solenoidal and irrotational.  
(OR)  
b) Prove that  $\nabla (r^n) = n r^{n-2} \vec{r}$ .
15. a) If  $F = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ , evaluate  $\iint (\nabla \times \vec{F}) \cdot d\vec{s}$  over the rectangle in the  $xy$  plane bounded by the lines  $x = 0, y = 0$ .  
(OR)  
b) Calculate  $\int_0^1 \int_0^1 \int_0^1 \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (y^2 + z^2)\vec{i} + (z^2 + x^2)\vec{j} + (x^2 + y^2)\vec{k}$  along the line joining these points.

**SECTION – C**

**(5 X 8 = 40 MARKS)**

**ANSWER EITHER ‘a’ OR ‘b’ IN EACH OF THE FOLLOWING QUESTIONS.**

16. a) Express  $\frac{\sin 6\theta}{\sin \theta}$  in terms of  $\cos \theta$ .  
(OR)  
b) Find the equation whose roots are  $2 \cos \frac{2\pi}{7}, 2 \cos \frac{4\pi}{7}, 2 \cos \frac{6\pi}{7}$

**(CONTD.....3)**

(3)

(16 UMS 203/ 16 UMA 203)

17. a) Separate into real and imaginary parts  $\tan^{-1} (x + iy)$ .  
(OR)

b) If  $\cos (x + iy) = r (\cos \alpha + i \sin \alpha)$ , prove that  $y = \frac{1}{2} \log \frac{\sin (x - \alpha)}{\sin (x + \alpha)}$ .

18. a) If  $f(x) = -x$  in  $-\pi < x < 0$   
 $= x$  in  $0 \leq x < \pi$

Expand  $f(x)$  as Fourier series in the interval  $-\pi$  to  $\pi$  of deduce  
that  $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$   
(OR)

b) Show that  $x^2 = \frac{x^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$  in the interval  $(-\pi \leq x \leq \pi)$ .

19. a) Prove that i)  $\operatorname{div} \operatorname{curl} \vec{F} = 0$ . ii)  $\operatorname{Curl} \operatorname{grad} \phi = 0$ .  
(OR)

b) Prove that  $\operatorname{div} (\vec{U} \times \vec{V}) = \vec{V} \cdot \operatorname{curl} \vec{U} - \vec{U} \cdot \operatorname{curl} \vec{V}$ .

20. a) Verify Stoke's theorem for  $\vec{F} = x^2 \vec{i} + z^2 \vec{j} + y^2 \vec{k}$  over the surface  $x + y + z = 1$  lying in the first octant.

(OR)

b) Verify Gauss divergence theorems for  $\vec{F} = y^2 \vec{i} + x^2 \vec{j} + z^3 \vec{k}$  over the cylindrical region  $x^2 + y^2 = 9$ ,  $z = 0$ ,  $z = 6$ .



A6.

(FOR THE CANDIDATES ADMITTED  
DURING THE ACADEMIC YEARS  
2015 AND 2016 ONLY)

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AIDED ..... 16 UMS 204 /15 UMS204  
S.F. .... 16 UMA 204/15 UMA204

NGM COLLEGE (AUTONOMOUS) :: POLLACHI

END – OF – SEMESTER EXAMINATIONS : MAY - 2017

B.Sc. - MATHEMATICS (AIDED S.F.)  
II SEMESTER

MAXIMUM MARKS: 75  
TIME: 3 HOURS

PART – III

ANALYTICAL GEOMETRY 2D AND 3D

SECTION – A

(10 X 1 = 10 MARKS)

ANSWER ALL QUESTIONS. CHOOSE THE BEST ANSWER.

- The distance between the points  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$  is given by -----  
a)  $PQ^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)$  b)  $PQ^2 = r_1^2 - r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)$   
c)  $PQ^2 = r_1^2 + r_2^2 - r_1 r_2 \cos(\theta_2 - \theta_1)$  d)  $PQ^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_1 - \theta_2)$
- If the lines  $\frac{x-1}{3} = \frac{y-2}{k} = \frac{z-3}{-2}$ ;  $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z-3}{-6}$  are perpendicular then the value of k is -----.  
a) -5 b) 5 c) 3 d) -2
- The centre of the sphere  $x^2 + y^2 + z^2 - 2x - 4y - 6z + 5$  is -----.  
a) (1, 1, 2) b) (2, 4, 1) c) (3, 1, 2) d) (1, 2, 3)
- A cone whose equation is of second degree is called a -----  
a) cone b) right circular cone c) quadric cone d) cylinder
- The equation of the right circular cone whose vertex is the origin, axis is the z axis and semi-vertical angle is  $\alpha$  is -----  
a)  $x^2 + y^2 = z^2 \tan^2 \alpha$  b)  $x^2 + y^2 = \tan^2 \alpha$  c)  $x^2 + y^2 = z^2 \tan \alpha$  d)  $x^2 + y^2 = z \tan^2 \alpha$
- Write the equation of circle in polar coordinates
- Define shortest distance between skew lines.
- Define power of a point with respect to a sphere.
- Write the equation of a quadric cone with its vertex at origin.
- Define right circular cylinder.

SECTION B

(5 X 5 = 25 MARKS)

ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS

11. (a) Derive the relationship between polar coordinates and Cartesian coordinates.

(OR)

11. (b) Find the equation of distance between two points  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$ .

12. (a) Find the equation of the line passing through the point (1, 1, 1) and parallel to the line whose

equation is  $\frac{x-2}{2} = \frac{2y+1}{3} = 3z - 1$ . (OR)

12. (b) Find the dir's of the line perpendicular to the lines  $x = \frac{y}{5} = -z$  and  $\frac{x}{2} = \frac{-y}{5} = \frac{z}{3}$ .

13. (a) Find the equation of the sphere whose centre is (6, -1, 2) and touches the plane  $2x - y + 2z = 2$ .

(OR)

13. (b) Find the equation of the tangent plane at the origin to the sphere  $x^2 + y^2 + z^2 + 8x - 6y + 4z = 0$ ;

14. (a) Derive the equation of a cone with vertex at origin.

(CONTD.....2)

(OR)

14. (b) Find the equation of the quadratic cone through the x, y, z axes and the lines through the origin with dir's 2, 4, 1; 3, 3, 1.
15. (a) Find the equation of the right circular cone whose vertex is the origin, axis is the line  $\frac{x}{1} = \frac{y}{2} = \frac{-z}{3}$  and semi vertical angle is  $30^\circ$ .

(OR)

15. (b) Show that the plane  $z = 0$  intersects the cone with vertex (2, 4, 1) which envelopes the sphere  $x^2 + y^2 + z^2 = 11$ , along a rectangular hyperbola.

**SECTION C**

**(5 X 8 = 40 MARKS)**

**ANSWER EITHER (a) OR (b) IN EACH OF THE FOLLOWING QUESTIONS.**

16. (a) Discuss the cases when tracing the conic  $\frac{l}{r} = 1 + e \cos \theta$ .

(OR)

16. (b) Trace the curve  $\frac{10}{r} = 3 \cos \theta + 4 \sin \theta + 5$ .

17. (a) Prove that the lines  $\frac{x+2}{2} = \frac{y+6}{3} = \frac{z-34}{-10}$  and  $\frac{x+6}{4} = \frac{y-7}{-3} = \frac{z-7}{-2}$  are coplanar. Find the point of intersection and the plane passing through them.

(OR)

17. (b) Find the S.D. and equation of the shortest distance between the lines:

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

18. (a) Find the equation of the sphere passing through the points (2, 3, 1); (5, -1, 2); (4, 3, -1) and (2, 5, 3).

(OR)

- (b) Find the equations of the planes which pass through the line  $x - 4y - z = 0$ ,  $x + z - 6 = 0$  and touch the sphere  $x^2 + y^2 + z^2 = 9$ .

19. (a) Find the equation of the cone whose vertex is the point (1, 1, 0) and whose guiding curve is  $x^2 + z^2 = 4$ ,  $y = 0$ .

(OR)

19. (b) Find the angle between the lines of intersection of the cone  $2xy - 2yz + zx = 0$  and the plane  $x + y - z = 0$ .

20. (a) If  $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$  represents a right circular cone, show that its axis is  $fx = gy = hz$ . and also find the semi vertical angle.

(OR)

20. (b) Find the equation of right circular cylinder whose radius is 2 and whose axis passes through (1, 2, 3) and has dir's 2, -3, 6.

A6/.

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