REAL ANALYSIS II
MULTIPLE CHOICE QUESTIONS

UNIT 1:

1. The function $f$ is continuous at $a \in M$ if $\lim_{x \to a} f(x) = \ldots$
   (a) $f(b)$
   (b) $f(c)$
   (c) $f(a)$ *
   (d) $f(x)$

2. The open ball of radius $r$ about $a$ is defined by ________
   (a) $B[r,a]$
   (b) $B[a,r]$ *
   (c) $B[-r,a]$
   (d) $B[-a,r]$

3. $\{X_n\}$ is a sequence in $M_1$ such that $\lim_{n \to \infty} X_n = \ldots$
   (a) $-a$
   (b) $0$
   (c) $1$
   (d) $a$ *

4. Every function from $\mathbb{R}^d$ is continuous on _______
   (a) $\mathbb{R}^d$ *
   (b) $-\mathbb{R}^d$
   (c) $\mathbb{R}^{2d}$
   (d) $\mathbb{R}^{3d}$

5. $<M,\rho>$, both $M$ and $\phi$ are _______
   (a) 1
   (b) Open sets *
   (c) 0
   (d) -1

6. Every subset of $\mathbb{R}^d$ is ______
   (a) open
   (b) closed
   (c) Open *
   (d) finite

7. Every open subset $G$ of $\mathbb{R}^d$ can be written as ______
   (a) 5
   (b) 3
   (c) open & closed
   (d) $G = \bigcup I_n$ *
8. $E$ is a closed subset of $M$ if $E = \ldots$
   (a) $E^*$
   (b) $W$
   (c) $G$
   (d) $U$

9. The set $\bar{E}$ of all limit points of $E$ is called the ______
   (a) open
   (b) Closure of $E^*$
   (c) open
   (d) compact

10. The subset $A$ of $M$ is said to be dense in $M$ if $\bar{A} =$_______
   (a) $N$
   (b) $L$
   (c) $M^*$
   (d) $U$

UNIT 2:

1. Every bounded subset of $\mathbb{R}^2$ is ______
   (a) Bounded
   (b) Not bounded
   (c) Totally bounded *
   (d) None

2. Every subsequence of a convergence sequence is __________
   (a) Divergent
   (b) Continuous
   (c) convergent *
   (d) Both (a) and (b)

3. Class of functions are called __________
   (a) contractions *
   (b) Distractions
   (c) Divergent
   (d) Convergent

4. $\{X_{nk}\}$ is a Cauchy subsequence of ______
   $k=1$
5. The Metric space \( <M, \rho> \) is both complete and totally bounded is said to be ________
   (a) scalar
   (b) complete
   (c) compact *
   (d) discrete

6. The space \( \mathbb{R}^d \) with finite subset is _________
   (a) discrete
   (b) complete
   (c) compact *
   (d) scalar

7. If \( M \) is a compact metric space then \( M \) has a __________
   (a) Heine Borel Property *
   (b) vector
   (c) scalar
   (d) mean value theorem

8. If \( F_1, F_2, \ldots, F_n \in \mathcal{F} \) then \( F_1 \cap F_2 \cap \ldots \cap F_n \neq _________
   (a) 1
   (b) 0
   (c) 2
   (d) \( \emptyset \) *

9. The real valued function \( f \) as continuous at the point \( a \in \mathbb{R} \) if given \( \varepsilon > 0 \) there exist \( \delta > 0 \) such that _________
   (a) 0
   (b) 1
   (c) \(|f(x) - f(a)| < \varepsilon \) *
   (d) \( \phi \)

10. If the real valued function \( f \) is continuous on the closed bounded interval \([a, b] \), then \( f \) is _________
    (a) Uniformly Continuous *
    (b) continuous
    (c) convergent
    (d) divergent
UNIT 3:

1. If \( \chi \) is a characteristic function of rational numbers \([0,1]\) then for any interval \( J \subset [0,1] \) then \( m[\chi,J]=\)___________
   
   (a) 0  *
   
   (b) \( \infty \)
   
   (c) \(-\infty\)
   
   (d) 1

2. If \( f \) is bounded function on the closed bounded interval \([a,b]\), we say that \( f \) is_________
   
   (a) Riemann – integral  *
   
   (b) Continuous
   
   (c) Bounded
   
   (d) None

3. \( \int_a^b f(x).dx=\int f(x).dx=\)___________
   
   (a) 1  *
   
   (b) 0
   
   (c) \( \infty \)
   
   (d) \(-\infty\)

4. \( \int_a^b f(x).dx=\)___________
   
   (a) \( \text{lub} \ U(\sigma,f) \)
   
   (b) \( \text{glb} \ U(\sigma,f) \)  *
   
   (c) \( \text{lub} \ U(\sigma,g) \)
   
   (d) \( \text{glb} \ U(\sigma,g) \)

5. \( \bigcup_{m=1}^\infty E_m=\)___________
   
   (a) \( E \)  *
   
   (b) \( M \)
   
   (c) \( E_n \)
   
   (d) \( 0 \)
6. $E^m \cup E^{**m} =$

(a) $E$
(b) $E^*$
(c) $\cap E^*$
(d) $UE^{**}$

7. $U[f, \tau] - L[f, \tau] <$

(a) d$|J|$
(b) a$|J|$
(c) b$|J|$
(d) c$|J|$

8. $\int f + \int f =$

(a) $\int f$
(b) $\int f^*$
(c) $\int f$
(d) $\int f$

9. If $f \in R(a,b)$, $g \in R(a,b)$ and $(f+g) \in R(a,b)$ then $\int (f+g) =$

(a) $\int f + \int g$
(b) $\int g + \int f$
(c) $\int f$
(d) $\int g$
10. Show that \( f(x) = 2x+1 \) is integrating on \([1,2]\) and \( \int (2x+1) \, dx = \) ________

  (a) 0
  (b) 2
  (c) 4 *
  (d) -4

UNIT 4:

1. \( g \) is continous function defined by \( g(x) = \)_______

  (a) \(< \cos x, \sin x> *\)
  (b) \(-<\cos x, \sin x>\)
  (c) \(<\cos x, -\sin x>\)
  (d) \(-<\cos x, -\sin x>\)

2. \( f \) is uniformly continous on \( R^1 \), if given \( \varepsilon > 0 \), there exists ______

  (a) \( \delta > 0 * \)
  (b) \( \delta > 1 \)
  (c) \( \delta < 1 \)
  (d) \( \delta < 0 \)

3. Subset \( B \) of \( R^2 \) consisting of graph of \( y=\sin(1/x) \), \( 0 < x \leq 1 \) together with the _____ interval on \( y \)-axis from \(<0,-1> \) to \(<0,1> \)

  (a) Open
  (b) Closed *
  (c) Continuous
  (d) Both (a) and (b)

4. Subset of \( R^2 \) is bounded, iff it is contained in some square whose edge has _____ length

  (a) infinite
  (b) finite *
  (c) none
  (d) both (a) and (c)
5. Subset of $\mathbb{R}^2$ is bounded, iff it is contained in some ______ whose edge has finite length

(a) Square*  
(b) rectangle  
(c) triangle  
(d) none

6. The interval _________ is not a bounded subset of $\mathbb{R}'$

(a) $(0,\infty)$*  
(b) $(\infty,0)$  
(c) $(0,1)$  
(d) $(1,\infty)$

7. Bounded and totally bounded are not at all _________

(a) non-equivalent  
(b) Equivalent*  
(c) finite  
(d) none

8. Since $\rho(e_j,e_k)=\sqrt{2},$ if $j \neq k$, the sequence $\rho_1, \rho_2, \ldots$ has no _________

(a) Cauchy’s sequence  
(b) Cauchy subsequence*  
(c) Convergent sequence  
(d) Divergent sequence

9. Metric space $[0,1]$ is _________ for $[0,1]$ is a closed subset of $\mathbb{R}'$

(a) Compact  
(b) Connect  
(c) Complete*  
(d) None

10. Metric space is denoted by _________

(a) $<\text{M}, \rho>$  
(b) $<\text{M}, \rho>^*$  
(c) $<\text{m}, \rho>$  
(d) $<\text{m}, \rho>$
UNIT5:

1. If \( f \) has a derivative at \( c \) and it is denoted by __________
   
   (a) \( f(c) \)
   (b) \( f'(c) \) *
   (c) \( f(1) \)
   (d) \( f'(0) \)

2. Define Rolle’s theorem.
3. Write down the statement of mean value theorem.
4. If \( f \) has a derivative at \( c \) then it is ________ at \( c \).
   
   (a) Neither or nor continuous
   (b) Bounded
   (c) Continous *
   (d) Both (a) and (b)

5. If \( f \) has derivative at \( c \) and \( g \) has derivative at \( f(c) \) then \( g \circ f \) has a ________ at \( c \).
   
   (a) Compact
   (b) Complete
   (c) Connectedness
   (d) Derivative *

6. If \( E \) is any subset of a metric space \( M \) then ________
   
   (a) \( \bar{E} \subseteq E \)
   (b) \( E \subseteq \bar{E} \) *
   (c) \( E \cap \bar{E} \)
   (d) \( \bar{E} \cap E \)

7. The union of a infinite number of closed sets need not be a ________
   
   (a) Closed set *
   (b) Open set
   (c) Both (a) and (b)
   (d) Union
8. If $A$ and $\phi$ are both open and closed in metric space $< A, \rho >$ then $A$ is said to be _______  

(a) Complete  
(b) Compact  
(c) Connected *  
(d) Closed  

9. If a subset $A$ of the metric space $< M, \rho >$ is totally bounded then $A$ is ________

(a) Unbounded  
(b) Bounded *  
(c) Continuous  
(d) Closed  

10. The space $R'$ is complete but not __________

(a) Connect  
(b) Continuous  
(c) Compact*  
(d) None
K2 QUESTIONS:

UNIT 1:

1. If a homeomorphism from $M_1$ onto $M_2$ exist, we say that $M_1$ and $M_2$ are
   \textbf{Answer: Homeomorphism}

2. Metric space $M$ is totally bounded if it has \_____________ sets.
   \textbf{Answer: finite number}

3. If $f$ is continuous at $a$, then $\omega[f; a] = \_____________
   \textbf{Answer: 0}$

4. If $f$ is not continuous at $a$, then $\omega[f; a] > \_____________
   \textbf{Answer: 0}$

5. Let $A = [0, 1]$, which of the following subsets of $A$ are open subset of $A$.
   \textbf{Answer: $(1/2, 1)$}

6. $\bar{A}_1 \cap A_2 = \_____________, A_1 \cap \bar{A}_2 = \phi$
   \textbf{Answer: $\phi$}

7. $x$ and $f$ are both continuous then $x \circ f$ is \_____________
   \textbf{Answer: Continuous *}

8. $A$ is not bounded we write $\text{diam } A = \_____________
   \textbf{Answer: $\infty$ *}$

9. If $T: M \rightarrow m$ is a contraction on $M$ then $\rho(t_x, t_y) \leq \__________$
   \textbf{Answer: $\alpha (\rho(x, y))$}

10. If every Cauchy sequence of sequence if points in $M$ converges to points in $M$ is called a
    \_____________
    \textbf{Answer: Complete metric space}

UNIT 2:

1. Function $f$ is bounded if its range $f(A)$ is a \_____________
   \textbf{Answer: Bounded subset}

2. If $f$ is a real valued function on a set $A$ that $f$ attains a maximum value of $a \in A$ if \__________
   \textbf{Answer: $f(a) \geq f(x), x \in A$}

3. If $f$ is a real valued function on a set $A$ that $f$ attains a minimum value of $a \in A$ if \__________
   \textbf{Answer: $f(a) \leq f(x), x \in A$}

4. A function is continuous if and only if it is uniformly continuous then it is said to be \_____________
   \textbf{Answer: Compact metric space}

5. The subset $E$ of $R'$ is said to be \_____________.
   \textbf{Answer: Measure zero}
6. \( \bigcup_{n=1}^{\infty} \mathbb{E}_n = \) _________

Answer: Measure zero

7. \( I_1 = [X, X_1], I_2 = [X_1, X_2], \ldots, I_n = [X_{n-1}, X_n] \) are called________

Answer: Component interval of \( \sigma \)

8. \( \mathbb{U}[f; \sigma] \geq \) ____________

Answer: \( \mathbb{L}(f, \sigma) \)

9. \( \int_a^b f(x) \, dx = \) _________

Answer: \( \text{lub} \, \mathbb{L}(f, \sigma) \)

10. \( \text{If } \chi \text{ is a characteristic function of relational numbers } [0, 1] \text{ then for any interval } J \subseteq [0, 1] \text{ then } M[\chi, J] = \) ____________

Answer: 1

UNIT 3:

1. If \( T^* \) is any refinement of \( T \), it may be show that \( \mathbb{L}[f; T] \leq \) ____________

Answer: \( \mathbb{L} \, [f; \, T^*] \)

2. \( \sum_{n=1}^{\infty} |I_n| \) converges to________

Answer: \( \bigcup_{n=1}^{b} \mathbb{E}_n \)

3. \( \int_a^b \lambda \, f = \) _________

Answer: \( \lambda \int_a^b \)

4. The function \( f \) is defined by \( f(x) = x^2 + 2x \), \( 0 < x < 4 \), then \( f(x) = \) ____________

Answer: 15
5. F1 and F2 are closed subsets of metric M, then F1UF2 is __________

   Answer: Closed

6. The union of an infinite number of closed sets is _________

   Answer: need not be a closed set

7. The set is open if and only if its complement is__________

   Answer: Closed

8. If f is continuous at a iff__________

   Answer: \( \lim_{n \to \infty} X_n = a \)

9. M is a metric space with_______ property

   (a) Heine borel

10. If \( |x-a| < \delta \), then the limit exceeds from_______

    Answer: \(-\infty < a < \infty\)

UNIT 4:

1. Metric space \(<M, \rho>\) the sets M and \(\phi\) are both ________

   Answer: Open and closed

2. If A is not bounded, then we write \(\text{diam} A\) equal to_______

   Answer: \(\infty\)

3. \( \cap \sum_{n=1}^{\infty} F_n \) contains________

   Answer: One point

4. The non-empty subsets A1,A2,……An of M exits such that__________

   Answer: \(\text{diam} A_k < 1\)
5. Which one is correct form of ‘contradiction’
   Answer: $\rho(tx,ty) \leq a(\rho(x,y))$

6. Choose the correct example for continuity of the inverse function
   Answer: $f(x) = x$

7. g is continuous function defined by
   Answer: $g(x) = \langle \cos x, \sin x \rangle$

8. Let $g$ be the continuous function defined by $g(x) = \langle \cos x, \sin x \rangle$, $0 \leq x \leq 2\pi$, then $g^{-1}$ is
   Answer: Continuous

9. $f$ is homomorphism of
   Answer: $m_1$ onto $m_2$

10. The space $\mathbb{R}^d$ with $\infty$ subset cannot be
    Answer: Compact

UNIT 5:

1. If $A$ is a closed subset of a compact metric space $<M, \rho>$ then $A$ is also
   Answer: Compact

2. If $M$ is a compact metric space then $M$ has a _______ property.
   Answer: Heine–Borel

3. If the metric space $M$ has a Heine-Borel property then $M$ is
   Answer: Compact

4. If $f$ has a derivative at $c$ then it is _______ at $c$.
   Answer: Continuous

5. If $f$ has derivative at $c$ and $g$ has derivative at $f(c)$ then $g \circ f$ has a _______ at $c$.
   Answer: Derivative

6. If $E$ is any subset of a metric space $M$ then
   Answer: E C Ė

7. The union of an infinite number of closed sets need not be a _______
   Answer: Closed set

8. If $A$ and $\phi$ are both open and closed in metric space $<A, \rho>$ then $A$ is said to be
   Answer: Connected

9. If a subset $A$ of the metric space $<M, \rho>$ is totally bounded then $A$ is _______
   Answer: Bounded

10. The space $\mathbb{R}'$ is complete but not _______
    Answer: Compact
K2 QUESTIONS:

Unit 1

1. If the real valued functions \( f \) and \( g \) are continuous at \( a \in \mathbb{R} \), then so are \( f + g, f - g \) and \( fg \). If \( g(a) \neq 0 \), then \( f/g \) is also continuous at ‘a’.
2. If \( f \) and \( g \) are real valued functions, if \( f \) is continuous at \( a \), and if \( g \) continuous at \( f(a) \), then \( g \circ f \) is continuous at ‘a’.
3. The real valued function \( f \) is continuous at \( a \in \mathbb{R} \) iff given \( \varepsilon > 0 \) there exist \( \delta > 0 \) such that \( |f(x) - f(a)| < \varepsilon, |x - a| < \delta \).
4. The real valued function \( f \) is continuous at \( a \in \mathbb{R} \) iff the inverse image under \( f \) of any open ball \( B[f(a), r] \) about \( f(a) \) contains a open ball \( B[a, \delta] \) about ‘a’.
5. The real valued function \( f \) is continuous at \( a \in \mathbb{R} \), iff whenever \( \{\epsilon_{n}\} \) is the sequence of real numbers convergent to ‘a’. Then the sequence \( \{f(\epsilon_{n})\} \) converges to \( f(a) \). \( \Rightarrow \lim_{n \to \infty} \epsilon_{n} = a \) \( \lim_{n \to \infty} f(\epsilon_{n}) = f(a) \)
6. The function \( f \) is continuous at \( a \in M \) if any one of the following condition holds
   (i) Given \( \varepsilon > 0 \) there exist \( \delta > 0 \) such that \( \rho_{2}(f(x), f(a)) < \varepsilon, \rho_{1}(x, a) < \delta \)
   (ii) The inverse image under \( f \) of any open ball \( B[f(a), \epsilon] \) about \( f(a) \) contains an open ball \( B[a, \delta] \) about ‘a’.
   (iii) Whenever \( Xn \xrightarrow{n} \) is a sequence of points in \( M \), converging to ‘a’.

Then the sequence \( Xn \xrightarrow{n} \) of points in \( M_{2} \) converges to \( f(a) \).

7. Let \( <M_{1}, \rho_{1}> \), \( <M_{2}, \rho_{2}> \) be metric spaces and let \( f:M_{1} \rightarrow M_{2} \) and \( g:M_{2} \rightarrow M_{3} \). If \( f \) is continuous at \( A \in M_{1} \) and \( g \) is continuous at \( f(a) \in M_{2} \). Then \( g(f) \) is continuous at \( A \).
8. If \( f \) and \( g \) are continuous function from a metric space \( M_{1} \) into a metric space \( M_{2} \) then so are \( f + g, f \cdot g \) and \( g(x) \neq 0, x \in M_{1} \).
9. Let \( \mathcal{G} \) be a non-empty family of open subsets of a metric space \( M \). Then \( U_{\mathcal{G}} \) is also in open subset of \( M \).
10. If \( G_{1} \) and \( G_{2} \) are open subsets of metric space \( M \) then \( G_{1} \cap G_{2} \) is also an open set.

Unit 2:

1. Let \( <M, \rho> \) be a metric space and let ‘A’ be a proper subset of \( M \) then the subset \( G \) of \( A \) is an open subset of metric space \( <A, \rho> \) iff there exists an open subset \( G_{M} \) of metric space \( <M, \rho> \) such that \( G_{A} = A \cap G_{M} \) (ie) A set is open in metric space \( <A, \rho> \) iff it is intersection of a set with ‘A’ that is open in metric space \( <M, \rho> \).
2. Let \( <M, \rho> \) be a metric space and Let \( A \subseteq M \), then if ‘a’ has either one of the following properties it has the other.
   i) Non empty subset \( A_{1} \) and \( A_{2} \) of \( M \) such that \( A = A_{1} U A_{2}, A \cap A_{2} = \phi, A_{1} \cap A_{2} = \phi \)
   ii) When \( <A, \rho> \) metric space then there is no set except \( A \) and \( \phi \) which is both open and closed in metric space \( <A, \rho> \). This we say that \( A \) is connected.
3. The subset of $A$ of $\mathbb{R}$ is connected iff whenever $a \in A$, $b \in A$ with $a < b$ then $C \in A$ for any $C$, such that $a < c < b$ that is whenever $a \in A$, $b \in A$, $a < b$, then $(a, b) \subseteq A$.

4. Let $F$ be a continuous function from metric space $M_1$. If $M_2$ is connected then the range of $F$ is connected.

5. Let $M$ be a metric space then $M$ is connected iff every continuous characteristic function on $M$, is constant $c$ (ie) $M$ is connected iff the function identically ‘zero’ and the function identically ‘1’ are the only characteristic functions on $M$ that are continuous on $M$.

6. If $A_1$ and $A_2$ be connected subsets of a metric space $M$ and if $A_1 \cap A_2 \neq \emptyset$ then $A_1 \cup A_2$ is also connected.

7. If the subset $A$ of the metric space $<M, \rho>$ is totally bounded then $A$ is bounded.

8. The subset $A$ of the metric space $<M, \rho>$ is totally bounded if and only if for every $\varepsilon > 0$, $A$ is a finite subset $\{x_1, \ldots, x_n\}$ which is $\varepsilon$ dense in $A$.

9. Let $<M, \rho>$ be a metric space, the subset $A$ of $M$ is totally bounded iff every sequence of points of $A$ contains a Cauchy subsequence.

Unit 3:

1. The metric space $<M, \rho>$ is compact iff every sequence of points in $M$ has a subsequence converging to a point in $M$.

2. If $A$ is a closed subset of the compact metric space $<M, \rho>$ then the metric space $<A, \rho>$ is also compact.

3. Let $A$ be a subset of a metric space $<M, \rho>$ is $<A, \rho>$ is compact, then $A$ is also closed subset of $<M, \rho>$.

4. If $M$ is a compact metric space then $M$ has a Heine Borel property.

5. If the metric space $M$ has a Heine Borel property then $M$ is compact.

6. The metric space $M$ is compact iff whenever $f$ is a family of closed subset of $M$ with the finite intersection property then $\bigcap_{F \in f} F \neq \emptyset$.

7. Let $f$ be a continuous function from compact metric space $M_1$, into the metric space $M_2$ (ie) $f$ is compact.

8. Let $f$ be a continuous function from the compact metric space $M_1 \to M_2$ then the range of $f$ is a bounded subset of $M_2$.

9. If the real valued function $f$ is continuous on closed bounded interval in $\mathbb{R}$ then $f$ must be bounded.

10. If the real valued function $f$ is continuous on the compact metric space $M$ then $f$ attains a maximum value at some point of $M$.

Unit 4:

1. If each of the subset $E_1$, $E_2$, $\ldots$ of $\mathbb{R}$ is of measure zero, then $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

2. If $f$ be a bounded function on $[a, b]$ then every upper sum for $f$ is greater than or equal to every lower sum of $f$ that is if $\sigma$ and $T$ are any two subdivisions of $[a, b]$ then $\sum_{i=1}^{n} f(x_i) \Delta x \geq L[f,T]$. 
3. Let f be a bounded function on the closed bounded interval \([a, b]\) then \(f \in \mathbb{R} [a, b]\) if and only if f is continuous at almost every point in \([a, b]\).

4. If \(\omega [f, x] x < a\) for each \(x\) in a closed bounded interval \(J\) then there is a subdivision \(\tau(J)\) such that \(U[f, \tau] - L[f, \tau] < a |J|\).

5. If \(f \in \mathbb{R}[a, b]\) and \(a < c < b\) then \(f \in \mathbb{R} [a, c]\), \(f \in \mathbb{R}[c, b]\) and, \(= +\)

6. If \(f \in \mathbb{R}[a, b]\) and \(\lambda\) is any real numbers then \(\lambda \in \mathbb{R} [a, b]\) and \(= \lambda\)

7. Every countable subset of \(\mathbb{R}\) as measure zero.

8. If \(f \in \mathbb{R}(a, b)\) and \(\lambda\) is any real number then \(\lambda \in \mathbb{R}(a, b)\) and \(= \lambda\)


10. STATE AND PROVE CHAIN RULE.

**Unit 5:**

1. Mean value theorem or Lagrange’s mean value theorem.

2. If \(f\) is a continuous real valued function on the interval \(J\) and if \(f'(x) > 0\) for all \(x\) in \(J\) except possibly the end point of \(J\) then \(F\) is strictly increasing on \(J\).

3. Let \(f\) and \(g\) be continuous functions on the closed bounded interval \([a, b]\) with \(g(a) f(b)\) if both \(f\) and \(g\) has derivative at each point of \((a, b)\) and \(f'(t)\) and \(g'(t)\) are not both equal to zero for any \(c \in (a, b)\) then there exist a point \(c \in (a, b)\) such that \(f'(c)/g'(c) = f(b)-f(a)/g(b)-g(a)\)

4. If \(f\) is a continuous on a closed bounded interval \([a, b]\) and if \(F(x)=\) at, \(a \leq x \leq b\)
   Then \(F(x)=f(x), a \leq x \leq b\)

5. If the real valued function \(f\) has the derivative at the point \(c \in \mathbb{R}\) then \(f\) is continuous at \(c\).

6. If \(f \in \mathbb{R} [a, b]\) if \(f(x) = \int_a^x f(t) dt a \leq x \leq b\) and if \(f\) is continuous at \(x, \in [a, b]\) then \(f'(x)=f(x)\).

7. Let \(f\) be a continuous real valued function on the closed bounded interval \([a, b]\). If the maximum value for \(f\) is attained at \(c\) where \(a < c < b\) and if \(f'(c)\) exists then \(f'(c) = 0\).

8. Let \(f\) be a continuous real valued function on the closed bounded interval \([a, b]\). If the minimum value of \(f\) is attained at \(c\) where \(a < c < b\) and if \(f'(c)\) exists then \(f'(c) = 0\).

9. If \(f'(x)=0\), for every \(x\) in the closed bounded interval \([a, b]\) then \(f\) is constant and closed interval \([a, b]\) \(f(x)=c, a \leq x \leq c\) for some \(c \in \mathbb{R}\).

10. IF \(f'(x)=g'(x)\) for all \(x\) in the closed bounded interval \([a, b]\) when \(f-g\) is constant i.e \(f(x)=g(x)+c\).
REAL ANALYSIS II

K3 QUESTIONS:

UNIT 1:

1. Every open set G of R' can be written G = U In where I1, I2, …… are mutually disjoint open intervals.
2. Let <M₁, ρ₁> and <M₂, ρ₂> be a metric spaces and let f: M₁ → M₂ then f is continuous on M if f⁻¹(G) is open in M₁ whenever G is open in M₂. That is f is continuous iff the inverse image of every open set in M₂ is open.
3. Let E be a subset of metric space M, then the point x ∈ M is a limit point of E iff every open ball B[x ; r] about x contains at least one point of E.
4. If F₁ and F₂ are closed subsets of metric M, then F₁ U F₂ is also closed.
5. If £ is a family of closed subsets of a metric space M then intersection or ∩ F is a closed set.

UNIT 2:

1. If <M, ρ> is a complete metric space, A is closed subset of M then <A, ρ> is also complete.
2. Let <M, ρ> be a complete metric space for n ∈ I. Let Fn be a closed bounded subset of M such that
   (i) F₁ ⊂ F₂ ⊂ …… ⊂ Fn ⊂ Fn+1 ⊂ ……
   (ii) Diam Fn → 0 as n → ∞ then n₀ Fn contains exactly one point.
3. Let <M, ρ> be a complete metric space if T is a contraction on M then, there is one and only one point x in one such that T x = x
4. The subset A of Rd is totally bounded iff it contains only a finite number of points.
5. If F is a continuous real valued function on the closed bounded interval [a, b] then f takes on every value between f(a) and f(b).

UNIT 3:

1. If the real valued function f is continuous on the closed bounded interval [a, b] then f attains the maximum or minimum values at point [a, b]
2. If f is a one to one continuous function from the compact metric space m₁ onto the metric space m₂, then f⁻¹ is continuous on m₂ and hence f is homomorphism of m₁ onto m₂
3. Let <M₁, ρ₁> be a compact metric space if f is continuous function from M₁ into M₂ a metric space <M₂, ρ₂> then f is uniformly continuous on m₁
4. If the real valued function is continuous on the closed bounded interval [a, b] then f is uniformly continuous on [a, b]
5. Let $<M_1, \rho_1>$ be a metric space and let $A$ be a dense subset of $M_1$. If $f$ is a uniformly continuous from $<A, \rho_1>$ into a complete metric space $<M_2, \rho_2>$ then $f$ can be extended to a uniformly continuous function $f$ from $M_1$ into $M_2$.

**UNIT 4:**

1. If $f \in \mathbb{R}(a, b)$, $g \in \mathbb{R}(a, b)$ and $(f+g) \in \mathbb{R}(a, b)$ then $\int f + \int g = \int (f+g)$

2. Properties of the Riemann integral

3. If $f \in \mathbb{R}[a, b]$, $g \in \mathbb{R}[a, b]$ and if $f(x) \leq g(x)$ almost everywhere ($a \leq x \leq b$) then $\int f \leq \int g$

4. If $f \in \mathbb{R}[a, b]$ then $\|f\| \in \mathbb{R}[a, b]$ and $|\int f| \leq \int |f|$

5. Let $f$ be a bounded function on the closed bounded interval $[a, b]$ then $f \in \mathbb{R}[a, b]$ if and only if for each $\varepsilon > 0$ there exist a subdivision $\sigma([a, b])$ such that $U(f, \sigma) < L(f, \sigma) + \varepsilon$

**UNIT 5:**

1. Second fundamental theorem of calculus

2. If $f$ and $g$ both have derivatives at $c \in \mathbb{R}'$ then $f+g$, $f-g, fg$ also have derivatives at $c \in \mathbb{R}'$ and $(f+g)'(c) = f'(c) + g'(c)$ and $(f-g)'(c) = f'(c) - g'(c)$ and $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$ further more if $g'(c) \neq 0$, then $f/g$ has a derivatives at $c$ and $(f/g)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{[g(c)]^2}$

3. Suppose $f$ has a derivative of $c$ and $g$ has a derivative at $f(c)$, then $\varphi = g \circ f$ has a derivative at $c$ and $\varphi'(c) = g'(f(c))f'(c)$

4. Let $\varphi$ be a real valued function on closed bounded interval $[a, b]$ such that $\varphi'$ is continuous on $[a, b]$. Let $A = \varphi(a)$, $B = \varphi(b)$ then if $f$ is continuous on $\varphi[a, b]$ we have $\int_{A}^{B} \varphi'(x) \, dx = \int_{a}^{b} f'(x) \, dx$

5. If $f$ has a derivative at every point of $[a, b]$. Then $f'$ takes an every value between $f'(a)$ and $f'(b)$